On the Complexity of Robust Stable Marriage

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Abstract. *Robust Stable Marriage (RSM)* is a variant of the classical *Stable Marriage* problem, where the robustness of a given stable matching is measured by the number of modifications required for repairing it in case an unforeseen event occurs. We focus on the complexity of finding an (a, b)-supermatch. An (a, b)-supermatch is defined as a stable matching in which if any a (non-fixed) men/women break up it is possible to find another stable matching by changing the partners of those a men/women and also the partners of at most b other couples. In order to show deciding if there exists an (a, b)-supermatch is $\mathcal{NP}$-complete, we first introduce a SAT formulation that is $\mathcal{NP}$-complete by using Schaefer's Dichotomy Theorem. Then, we show the equivalence between the SAT formulation and finding a $(1, 1)$-supermatch on a specific family of instances.

1 Introduction

Matching under preferences is a multidisciplinary family of problems, mostly studied by the researchers in the field of economics and computer science. There are many variants of the matching problems such as College Admission, Hospital/Residents, Stable Marriage, Stable Roommates, etc. The reader is referred to the book written by Manlove for a comprehensive background on the subject [1].

We work on the robustness notion of stable matching proposed by Genc et. al. [2]. In the context of Stable Marriage, the purpose is to find a matching M between men and women such that no pair $\langle man, woman \rangle$ prefer each other to their situations in M. The authors of [2] introduced the notion of (a, b)-supermatch as a measure of robustness. An (a, b)-*supermatch* is a stable matching such that if any a agents (men or woman) break up it is possible to find another stable matching by changing the partners of those a agents with also changing the partners of at most b other couples. However, they leave the complexity of this problem open [2].

The focus of this paper is to study the complexity of finding an (a, b)-supermatch. In order to show that the general case of RSM, which is the decision of existence of an (a, b)-supermatch, is $\mathcal{NP}$-complete, it is sufficient to show that a restricted version of the general problem is $\mathcal{NP}$-complete. Thus, we first show

that the decision problem for finding a $(1,1)$-supermatch on a restricted family of instances is $\mathcal{NP}$-complete, then we generalize this complexity result to the general case. Proofs and details in this paper are mostly omitted due to space restrictions. The details can be found in our technical paper [3].

2 Notations & Background

An instance of the *Stable Marriage problem (with incomplete lists)* takes as input a set of men $U = \{m_1, m_2, \ldots, m_{n_1}\}$ and a set of women $W = \{w_1, w_2, \ldots, w_{n_2}\}$ where each person has an ordinal preference list over members of the opposite sex. For the sake of simplicity we suppose in the rest of the paper that $n_1 = n_2$. A *pair* $\langle m_i, w_j \rangle$ is acceptable if w_j (respectively m_i) appears in the preference list of m_i (respectively w_j). A matching is a set of acceptable pairs where each man (respectively woman) appears at most once in any pair of M. If $\langle m_i, w_j \rangle \in M$, we say that w_j (respectively m_i) is the partner of m_i (respectively w_j) and then we denote $M(m_i) = w_j$ and $M(w_j) = m_i$. A pair $\langle m_i, w_j \rangle$ (sometimes denoted as $\langle i, j \rangle$) is said to be *blocking* a matching M if m_i prefers w_j to $M(m_i)$ and w_j prefers m_i to $M(w_j)$. A *matching M* is called *stable* if there exists no blocking pair for M. A *pair* $\langle m_i, w_j \rangle$ is said to be *stable* if it appears in a stable matching. A pair $\langle m_i, w_j \rangle$ is called *fixed* if $\langle m_i, w_j \rangle$ appears in every stable matching. In this case, the man m_i and woman w_j are called fixed. In the rest of the paper we use n to denote the number of non-fixed men and $\mathcal{I}$ be an instance of a Stable Marriage problem. We measure the distance between two stable matchings M_i, M_j by the number of men that have different partners in M_i and M_j, denoted by $d(M_i, M_j)$.

Formally, a stable matching M is said to be (a, b)-supermatch if for any set $\Psi \subset M$ of a stable pairs that are not fixed, there exists a stable matching M' such that $M' \cap \Psi = \emptyset$ and $d(M, M') - a \leq b$ [2].

Definition 1 (π_1) *INPUT: $a, b \in \mathbb{N}$, and a Stable Marriage instance $\mathcal{I}$.*
QUESTION: Is there an (a, b)-supermatch for $\mathcal{I}$?

Let M be a stable matching. A ***rotation*** $\rho = (\langle m_{k_0}, w_{k_0} \rangle, \langle m_{k_1}, w_{k_1} \rangle, \ldots, \langle m_{k_{l-1}}, w_{k_{l-1}} \rangle)$ (where $l \in \mathbb{N}^*$) is an ordered list of pairs in M such that changing the partner of each man m_{k_i} to the partner of the next man $m_{k_{i+1}}$ (the operation $+1$ is modulo l) in the list ρ leads to a stable matching denoted by M/ρ. The latter is said to be obtained after *eliminating* ρ from M. In this case, we say that $\langle m_{l_i}, w_{l_i} \rangle$ is *eliminated* by ρ, whereas $\langle m_{l_i}, w_{l_{i+1}} \rangle$ is *produced* by ρ, and that ρ is *exposed* on M. If a pair $\langle m_i, w_j \rangle$ appears in a rotation ρ, we denote it by $\langle m_i, w_j \rangle \in \rho$. Additionally, if a man m_i appears at least in one of the pairs in the rotation ρ, we say m_i is *involved* in ρ. There exists a partial order for rotations. A rotation ρ' is said to precede another rotation ρ (denoted by $\rho' \prec\prec \rho$), if ρ' is eliminated in every sequence of eliminations that starts at M_0 and ends at a stable matching in which ρ is exposed [4]. Note that this relation is transitive, that is, $\rho'' \prec\prec \rho' \wedge \rho' \prec\prec \rho \implies \rho'' \prec\prec \rho$. Two rotations are said to be *incomparable* if one does not precede the other.

The structure that represents all rotations and their partial order is a directed graph called ***rotation poset*** denoted by $\Pi = (\mathcal{V}, E)$. Each rotation corresponds to a vertex in $\mathcal{V}$ and there exists an edge from ρ' to ρ if ρ' precedes ρ. There are two different edge types in a rotation poset: **type 1** and **type 2**. Suppose $\langle m_i, w_j \rangle$ is in rotation ρ, if ρ' is the unique rotation that moves m_i to w_j then $(\rho', \rho) \in E$ and ρ' is called a type 1 predecessor of ρ. If ρ moves m_i below w_j, and $\rho' \neq \rho$ is the unique rotation that moves w_j above m_i, then $(\rho', \rho) \in E$ and ρ' is called a type 2 predecessor of ρ [4]. A node that has no outgoing edges is called a *leaf node* and a node that has no incoming edges is called *root node*.

A ***closed subset*** S is a set of rotations such that for any rotation ρ in S, if there exists a rotation ρ' that precedes ρ then ρ' is also in S. Every closed subset in the rotation poset corresponds to a stable matching [4]. Let $\mathrm{L}(S)$ be the set of rotations that are the leaf nodes of S. Similarly, let $\mathrm{N}(S)$ be the set of the rotations that are not in S, but all of their predecessors are in S. This can be illustrated as having a cut in the graph Π, where the cut divides Π into two sub-graphs, namely Π_1 and Π_2. If there are any comparable nodes between Π_1 and Π_2, Π_1 is the part that contains the preceding rotations. Eventually, Π_1 corresponds to the closed subset S, $\mathrm{L}(S)$ corresponds to the leaf nodes of Π_1 and $\mathrm{N}(S)$ corresponds to the root nodes of Π_2.

Let us illustrate these terms on a sample SM instance specified by the preference lists of 7 men/women in Table 1 given by Genc et. al [2]. For the sake of clarity, each man m_i is denoted with i and each woman w_j with j. Figure 2 represents the rotation poset and all the rotations associated with this sample.

m_0	0 6 5 2 4 1 3	w_0	2 1 6 4 5 3 0
m_1	6 1 4 5 0 2 3	w_1	0 4 3 5 2 6 1
m_2	6 0 3 1 5 4 2	w_2	2 5 0 4 3 1 6
m_3	3 2 0 1 4 6 5	w_3	6 1 2 3 4 0 5
m_4	1 2 0 3 4 5 6	w_4	4 6 0 5 3 1 2
m_5	6 1 0 3 5 4 2	w_5	3 1 2 6 5 4 0
m_6	2 5 0 6 4 3 1	w_6	4 6 2 1 3 0 5

Table 1. Preference lists for men (left) and women (right) for a sample instance of size 7.
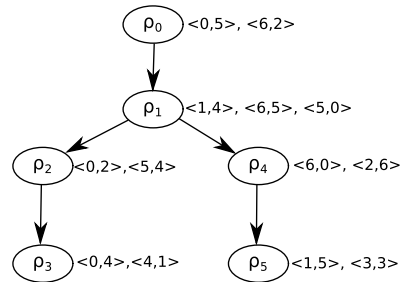
Table 2. Rotation poset of the instance given in Table 1.

In this example, $M_1 = \{(0,2), (1,4), (2,6), (3,3), (4,1), (5,0), (6,5)\}$ is a stable matching. The closed subset $S_2 = \{\rho_0, \rho_1\}$ corresponds to $M_1/\rho_1 = M_2 = \{(0,2), (1,5), (2,6), (3,3), (4,1), (5,4), (6,0)\}$. For M_2, leaf and neighbor nodes can be identified as $\mathrm{L}(S_2) = \{\rho_1\}$ and $\mathrm{N}(S_2) = \{\rho_2, \rho_4\}$.

3 A specific problem family

In this section, we describe a restricted, specific family F of Stable Marriage instances over properties on its generic rotation poset $\Pi_F = (\mathcal{V}_F, E_F)$.

Property 1 Each rotation $\rho_i \in \mathcal{V}_F$, contains exactly 2 pairs $\rho_i = (\langle m_{i1}, w_{i1} \rangle,$
$\langle m_{i2}, w_{i2} \rangle)$.
Property 2 Each rotation $\rho_i \in \mathcal{V}_F$, has at most 2 predecessors and 2 successors.
Property 3 Each edge $e_i \in E_F$, is a type 1 edge.
Property 4 For each man $m_i, i \in [1, n]$, m_i is involved in at least 2 rotations.

Lemma 1. *For each two different paths P_1 and P_2 defined on Π_F, where both start at rotation ρ_s, end at ρ_t, and the pair $\langle m_e, w_f \rangle \in \rho_s$, if all rotations on P_1 (respectively P_2) contain m_e, at least one of the rotations on P_2 (respectively P_1) does not contain w_f.*

Definition 2 (π_1^F) *A particular case of π_1, with the restrictions from problem family* F.

In order to prove that the general problem π_1 is $\mathcal{NP}$-complete, we first show that the restricted family problem π_1^F is $\mathcal{NP}$-complete. In order to do this, we prove it for a particular case noted π_2^F.

Definition 3 (π_2) *The special case of π_1, where $a = 1, b = 1$.*

Definition 4 (π_2^F) *INPUT: A Stable Marriage instance $\mathcal{I}$ from family* F.
QUESTION: Is there a $(1, 1)$-supermatch for $\mathcal{I}$?

4 Complexity results

In order to show that π_2^F is $\mathcal{NP}$-complete, we first need to define a particular SAT problem denoted by SAT-SM which is $\mathcal{NP}$-complete.

SAT-SM takes as input a set of integers $\mathcal{X} = [1, |\mathcal{X}|]$, n lists $l_1, l_2, \ldots, l_n$ where $n \in \mathbb{N}^*$ and each l_a $(a \in [1, n])$ is an ordered list of integers of $\mathcal{X}$, and three sets of distinct Boolean variables $Y = \{y_e \mid e \in \mathcal{X}\}$, $S = \{s_e \mid e \in \mathcal{X}\}$, and $P = \{p_e \mid e \in \mathcal{X}\}\}$.

Conditions on the lists: The lists $l_1, \ldots l_n$ as subject to the following constraints: First, $\forall a \in [1, n]$, l_a is denoted by $(\mathcal{X}_1^a, \ldots, \mathcal{X}_{k_{l_a}}^a)$, where $k_{l_a} = |l_a| \geq 2$. Second, each element of $\mathcal{X}$ appears in exactly two different lists. For illustration, the set $\mathcal{X}$ represents the indexes of rotations and a list l_a represents the index of each rotation having the man m_a. The order in l_a specifies the path in the rotation poset from the first rotation to the last one for a man m_a. And the restriction for having each index in two different lists is related to Property 1.

In addition to those two conditions, we have the following rule over the lists: **[Rule 1]** For any $\mathcal{X}_i^m$ and $\mathcal{X}_j^m$ from the same list l_m where $m \in [1, n]$ and $j > i$, there does not exist any sequence S that starts at $\mathcal{X}_i^m$ and ends at $\mathcal{X}_j^m$ constructed by iterating the two consecutive rules σ and θ below:

σ) given $\chi_e^a \in S$, the next element in S is χ_{e+1}^a, where $e + 1 \leq k_{l_a}$.

θ) given $\chi_e^a \in S$, the next element in S is χ_f^b, where $\chi_e^a = \chi_f^b, a \neq b \in [1, n]$, and $1 \leq f \leq k_{l_b}$.

Conditions on the clauses: The CNF that defined SAT-SM is a conjunction of four groups of clauses: (A), (B), (C) and (D). The groups are subject to the following conditions:

(A): For any list $l_a, a \in [1, n]$, $(\chi_1^a, \ldots, \chi_{k_{l_a}}^a)$, we have a disjunction between the Y-elements and the P-elements as $\bigwedge_{l=1}^{n} \left(\bigvee_{i=1}^{k_l} y_{\chi_i^a} \vee p_{\chi_i^a} \right)$.

(B): For any list $l_a, a \in [1, n]$, $(\chi_1^a, \ldots, \chi_{k_{l_a}}^a)$, we have a disjunction between two S-elements with consecutive indexes defined by $\bigwedge_{i=1}^{k_{l_a}-1} s_{\chi_i^a} \vee \neg s_{\chi_{i+1}^a}$.

(C): This group of clauses is split in two. For any list $l_a, a \in [1, n]$, $(\chi_1^a, \ldots, \chi_{k_{l_a}}^a)$, the first sub-group C_1 contains all the clauses defined by the logic formula $\bigwedge_{i=1}^{k_{l_a}-1} y_{\chi_i^a} \rightarrow s_{\chi_i^a} \wedge \neg s_{\chi_{i+1}^a}$. With a CNF notation, it leads to $\bigwedge_{i=1}^{k_{l_a}-1}(\neg y_{\chi_i^a} \vee s_{\chi_i^a}) \wedge (\neg y_{\chi_i^a} \vee \neg s_{\chi_{i+1}^a})$.

The second sub-group C_2 has three specific cases according to the position of elements in the ordered lists. As fixed above, each element of χ appears in exactly two different lists. Thus, for any $e \in \chi$, there exists two lists l_a and l_b such that $\chi_i^a = \chi_j^b = e$, where $i \in [1, k_{l_a}]$ and $j \in [1, k_{l_b}]$. For each couple of elements of χ denoted by (χ_i^a, χ_j^b) that are equal to the same value e, we define a clause with these elements and the next elements in their lists respecting the ordering: $s_{\chi_i^a} \rightarrow y_{\chi_i^a} \vee s_{\chi_{i+1}^a} \vee s_{\chi_{j+1}^b}$. With a CNF notation it leads to: $(\neg s_{\chi_i^a} \vee y_{\chi_i^a} \vee s_{\chi_{i+1}^a} \vee s_{\chi_{j+1}^b})$.

(D): Similarly, for each couple of elements of χ denoted by (χ_i^a, χ_j^b) equal to the same value e, we define a clause with these elements and the previous elements in their lists respecting the ordering: $p_{\chi_i^a} \leftrightarrow \neg s_{\chi_i^a} \wedge s_{\chi_{i-1}^a} \wedge s_{\chi_{j-1}^b}$. With a CNF notation, it leads to:

$(\neg p_{\chi_i^a} \vee \neg s_{\chi_i^a}) \wedge (\neg p_{\chi_i^a} \vee s_{\chi_{i-1}^a}) \wedge (\neg p_{\chi_i^a} \vee s_{\chi_{j-1}^b}) \wedge (s_{\chi_i^a} \vee \neg s_{\chi_{i-1}^a} \vee \neg s_{\chi_{j-1}^b} \vee p_{\chi_i^a})$

To conclude the definition, the full CNF formula of SAT-SM is (A) $\wedge$ (B) $\wedge C_1 \wedge C_2 \wedge$ (D).

The SAT-SM problem is the question of finding an assignment of the Boolean variables that satisfies the above CNF formula.

Theorem 1. *The SAT-SM problem is $\mathcal{NP}$-complete.*

Proof. SAT-SM is $\mathcal{NP}$-complete by using Schaefer's dichotomy theorem [5]. Details of the full proof can be found in the technical paper [3].

Theorem 2. *The decision problem π_2^F is $\mathcal{NP}$-complete.*

Proof. The verification is shown to be polynomial-time decidable [2]. Therefore, π_2^F is in $\mathcal{NP}$. We show that π_2^F is $\mathcal{NP}$-complete by presenting a polynomial reduction from the SAT-SM problem to π_2^F as follows.

From an instance $\mathcal{I}_{SSM}$ of SAT-SM, we construct in polynomial time an instance $\mathcal{I}$ of $\boldsymbol{\pi_2^F}$. This means the construction of the rotation poset $\Pi_F = (\mathcal{V}_F, E_F)$ with all stable pairs in the rotations, and the preference lists.

We first start constructing the set of rotations $\mathcal{V}_F$ and then proceed by deciding which man is a part of which stable pair in which rotation. First, $\forall e \in \mathcal{X}$, we have a corresponding rotation ρ_e. Second, $\forall l_a, a \in [1, n], \forall \mathcal{X}_i^a \in [1, k_{l_a}]$, we insert m_a as the man to the first empty pair in rotation $\rho_{\mathcal{X}_i^a}$. Each man of $\boldsymbol{\pi_2^F}$ is involved in at least two rotations (satisfying Property 4).

As each $\mathcal{X}_i^a$ appears in exactly two different lists l_a and l_b, each rotation is guaranteed to contain exactly two pairs involving different men m_a, m_b (Property 1), and to possess at most two predecessors and two successors in Π_F (Property 2).

For the construction of the set of arcs E_F, for each couple of elements of $\mathcal{X}$ denoted by $(\mathcal{X}_i^a, \mathcal{X}_{i+1}^a)$, $a \in [1, n], \forall i \in [1, k_{l_a} - 1]$, we add an arc from $\rho_{\mathcal{X}_i^a}$ to $\rho_{\mathcal{X}_{i+1}^a}$. Note that this construction, yields in each arc in E representing a type 1 relationship (Property 3).Because each arc links two rotations, where exactly one of the men is involved in both rotations. Now, in order to complete the rotation poset Π_F, the women involved in rotations must also be added. The following procedure is used to complete the rotation poset:

1. For each element $\mathcal{X}_1^a \in \mathcal{X}$, with $a \in [1, n]$, let $\rho_{\mathcal{X}_1^a}$ be the rotation that involves man m_a. In this case, the partner of m_a in $\rho_{\mathcal{X}_1^a}$ is completed by inserting woman w_a, so that the resulting rotation contains the stable pair $\langle m_a, w_a \rangle \in \rho_{\mathcal{X}_1^l}$.
2. We perform a breadth-first search on the rotation poset from the completed rotations. For each complete rotation $\rho = (\langle m_i, w_b \rangle, \langle m_k, w_d \rangle) \in \mathcal{V}_F$, let ρ_{s1} (resp. ρ_{s2}) be one of the successor of ρ and modifying m_i (resp. m_k). If ρ_{s1} exists, then we insert the woman w_d in ρ_{s1} as the partner of man m_i. In the same manner, if ρ_{s2} exists, we insert the woman w_b in ρ_{s2} as the partner of man m_k. The procedure creates at most two stable pairs. From the fact that each woman w_b appears in the next rotation as partnered with the next man of the current rotation ρ, in the SAT-SM definition it is equivalent to going from $\mathcal{X}_y^i$ to $\mathcal{X}_{z+1}^k$ on lists where $\mathcal{X}_y^i = \mathcal{X}_z^k, y \in [1, [\,]n], z \in [1, n-1]$. Thus the path where the woman appears follow a sequence defined as the one in [**Rule 1**] from the SAT-SM definition. By this rule, we can conclude that Lemma 1 is satisfied.

All along the construction, we showed that all the properties required, to have a valid rotation poset from the family F, are satisfied. Using this process we are adding equal number of women and men in the rotation poset.

The last step to obtain an instance $\mathcal{I}$ of $\boldsymbol{\pi_2^F}$ is the construction of the preference lists. By using the rotation poset created above, we can construct incomplete preference lists for the men and women. We use a similar approach to a procedure previously defined by Gusfield et. al. for creating the lists [6]:

– Apply topological sort on $\mathcal{V}_F$.

- For each man $m_i \in [1, n]$, insert woman w_i as the most preferred to m_i's preference list.
- For each woman $w_i \in [1, n]$, insert man m_i as the least preferred to w_i's preference list.
- For each rotation $\rho \in \mathcal{V}_F$ in the ordered set, for each pair $\langle m_i, w_j \rangle$ produced by ρ, insert w_j to the man m_i's list in decreasing order of preference ranking. Similarly, place m_i to w_j's list in increasing order of preference ranking.

The Lemma 1 imposed on our rotation poset clearly involves that each preference list contains each member of the opposite sex at most once. To finish, one can observe that the instance obtained respects the Stable Marriage requirements and the specific properties from problem family F.

$\Leftarrow$ Suppose that there exists a solution to an instance $\mathcal{I}$ of the decision problem π_2^F. Then we have a $(1,1)$-supermatch and its corresponding closed subset S. As defined in Section 2, $L(S)$ is the set of leaf nodes of S, $N(S)$ the set of nodes such that all their predecessors are in S but not themselves. From these two sets, we can assign all the literals in $\mathcal{I}_{SSM}$ as follows:

- For each rotation $\rho_i \in L(S)$, set $y_i = true$. Otherwise, set $y_i = false$.
- For each rotation $\rho_i \in S$, set $s_i = true$. Otherwise, set $s_i = false$.
- For each rotation $\rho_i \in N(S)$, set $p_i = true$. Otherwise, set $p_i = false$.

If S represents a $(1,1)$-supermatch, that means by removing only one rotation present in $L(S)$ or by only adding one rotation from $N(S)$, any pair of the corresponding stable matching can be repaired with no additional modifications. Thus any men must be contained in a leaf or a neighbor node. This leads to having for each man one of the literals assigned to true in his list in SAT-SM. Therefore every clause in (A) are satisfied.

For the clauses in (B), for any man's list the clauses are forcing each s_i literal to be true if the next one s_{i+1} is. By definition of a closed subset, from any leaf of S, all the preceding rotations (indexes in the lists) must be in S. And thus every clause in (B) is satisfied.

As the clauses in (C) altogether capture the definition of being a leaf node of S, they are all satisfied by $L(S)$. At last, for the clauses in (D), it is also easy to see that any rotation being in $N(S)$ is equivalent to not being in the solution and having predecessors in. Thus all the clauses are satisfied.

Thus we can conclude that this assignment satisfy the SAT formula of $\mathcal{I}_{SSM}$.

$\Rightarrow$ Suppose that there exists a solution to an instance $\mathcal{I}_{SSM}$ of the decision problem SAT-SM. Thus we have a valid assignment to satisfy the SAT formula of $\mathcal{I}_{SSM}$. We construct a closed subset S to solve $\mathcal{I}$. As previously, we use the sets $L(S)$ and $N(S)$, then for each literal y_i assigned to true, we put the rotation ρ_i in $L(S)$. We are doing the same for p_i and s_i as above.

The clauses in (B) enforce the belonging to S of all rotations preceding any element of S, thus the elements in S form a closed subset. To obtain a $(1,1)$-supermatch, we have to be sure we can repair any couple by removing only one

rotation present in L(S) or by only adding one rotation from N(S). The clauses in Ⓒ enforce the rotations in $L(S)$ to be without successors in S. And in the same way the clauses in Ⓓ enforce the rotations in $N(S)$ to not be in S but have their predecessors in the solution.

Now we just have to check that all the men are contained in at least one rotation from L(S) ∪ N(S). By the clauses from Ⓐ, we know that at least one y_e or p_e for any man m_i is assigned to true. Thus from this closed subset S, we can repair any couple $\langle m_i, w_j \rangle$ in one modification by removing/adding the rotation having m_i. Since there exists a $1-1$ equivalence between a stable matching and the closed subset in the rotation poset, we have a $(1, 1)$-supermatch. □

Corollary 1 *From the Theorem 2 and by generality, both decision problems $\boldsymbol{\pi_1}$ and $\boldsymbol{\pi_2}$ are $\mathcal{NP}$-complete.*

5   Concluding Remarks

We study the complexity of the Robust Stable Marriage (RSM) problem. In order to show that given a Stable Marriage instance, deciding if there exists an (a, b)-supermatch is $\mathcal{NP}$-complete, we first introduce a SAT formulation which models a specific family of Stable Marriage instances. We show that the formulation is $\mathcal{NP}$-complete by Schaefer's Dichotomy Theorem. Then we apply a reduction from this problem to prove the $\mathcal{NP}$-completeness of RSM.

6   Acknowledgements

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