Centre for
Data Analytics

Insight

Revisiting Two-Sided Stability Constraints

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May 18, 2016

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Context

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- Not much about local consistency levels
- Global constraints for stable matching problems?

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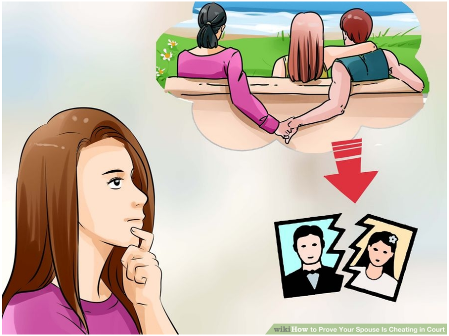
Stable Marriage [Gale and Shapley, 1962]



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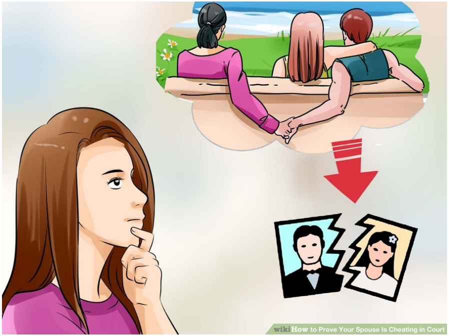
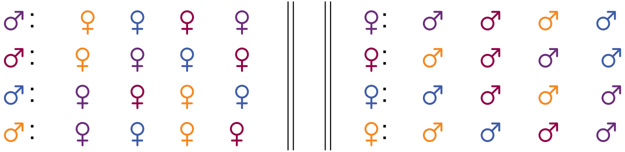


- ♂, ♂, ♂, . . .
- ♀, ♀, ♀, . . .
- Two sided preferences
- A matching *M* is stable when no blocking pair exists
- A pair (♂,♀) is blocking a matching M if ♂/♀ prefer each other to their situation in *M*

[Gale and Shapley, 1962]



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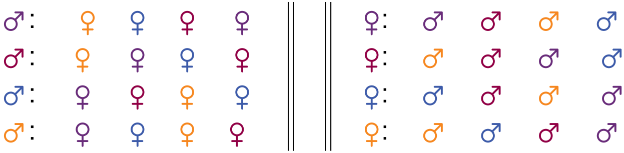

- Male proposals

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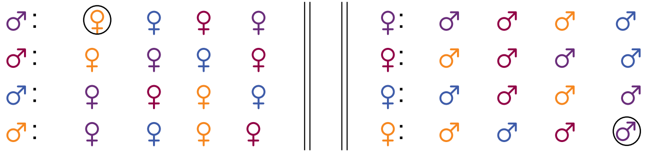

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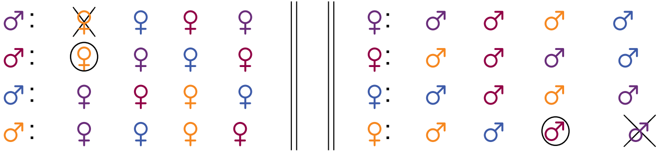

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  (♂,♀), (♂,♀)

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♂:	~~♀~~	♀	♀	♀	‖ ‖	♀:	♂	♂	♂	♂
♂:	♀	♀	♀	♀	‖ ‖	♀:	♂	♂	♂	♂
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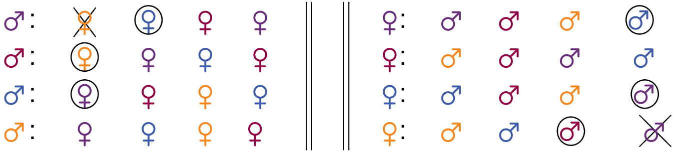

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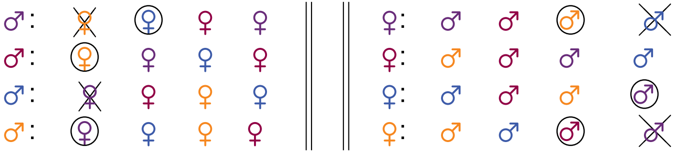

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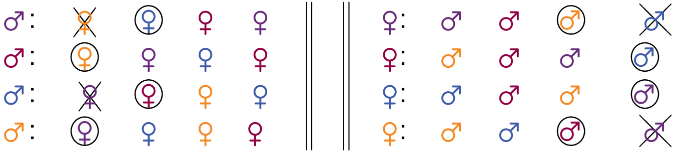

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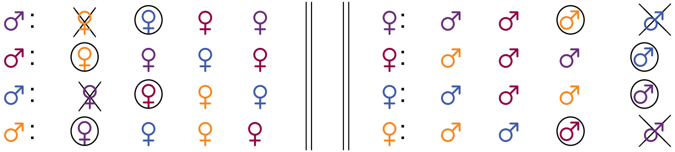

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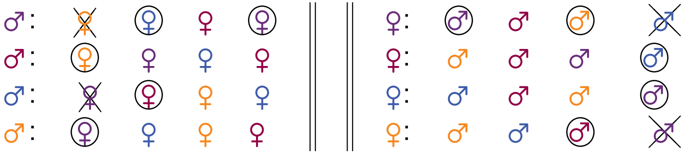

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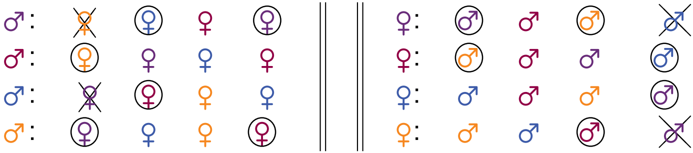

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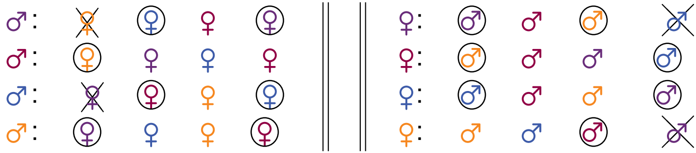

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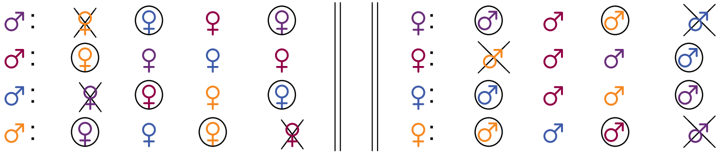

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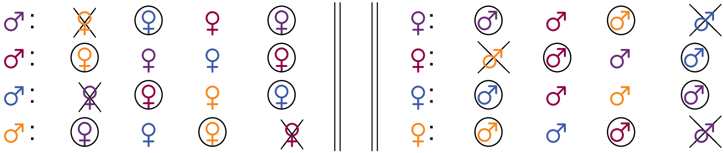

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- Find a stable matching?

Previous CP Approaches

- Some global constraints exist in the literature [Gent et al., 2001, Unsworth and Prosser, 2005, Manlove et al., 2007, Unsworth and Prosser, 2013]

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- They are all equivalent in terms filtering (even for SM)
- They all are equivalent to the GL-Lists
- What is the level of consistency?

CP Model for Hospital/Resident Problem (Γ)

Variables

- x_i: index of the hospital assigned to r_i
- $y_{j,k}$: index of the resident assigned to the k^{th} position in h_j

Constraints

$$y_{j,k} < y_{j,k+1} \; (\forall j \in [1, n_H], \forall k \in [1, c_j - 1]) \tag{1}$$

$$y_{j,k} \geq q_{i,j} \implies x_i \leq p_{i,j} \; (\forall j \in [1, n_H], \forall k \in [1, c_j], \forall i \in \mathcal{H}_j) \tag{2}$$

$$x_i \neq p_{i,j} \implies y_{j,k} \neq q_{i,j} \; (\forall i \in [1, n_R], \forall j \in \mathcal{R}_i, \forall k \in [1, c_j]) \tag{3}$$

$$(x_i \geq p_{i,j} \wedge y_{j,k-1} < q_{i,j}) \implies y_{j,k} \leq q_{i,j} \; (\forall i \in [1, n_R], \forall j \in \mathcal{R}_i, \forall k \in [1, c_j]) \tag{4}$$

$$y_{j,c_j} < q_{i,j} \implies x_i \neq p_{i,j} \; (\forall j \in [1, n_H], \forall i \in \mathcal{H}_j) \tag{5}$$

Example

$$
\begin{array}{ll|l}
\mathcal{R}_1 = [3,2,1] & & \mathcal{H}_1 = [1,2,4] \\
\mathcal{R}_2 = [4,1,3,2] & & \mathcal{H}_2 = [2,1,3] \\
\mathcal{R}_3 = [2,4,3] & & \mathcal{H}_3 = [3,2,4,1] \\
\mathcal{R}_4 = [1,3,4] & & \mathcal{H}_4 = [4,3,2]
\end{array}
$$

Initial Domain

- $\mathcal{D}(x_1) = \mathcal{D}(x_3) = \mathcal{D}(x_4) = \{1,2,3,5\}$
- $\mathcal{D}(x_2) = \{1,2,3,4,5\}$
- $\mathcal{D}(y_{1,0}) = \mathcal{D}(y_{2,0}) = \mathcal{D}(y_{3,0}) = \mathcal{D}(y_{4,0}) = \{0\}$
- $\mathcal{D}(y_{1,1}) = \mathcal{D}(y_{2,1}) = \mathcal{D}(y_{4,1}) = \{1,2,3,5\}$
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2-SidedStability$(\mathcal{X}, \mathcal{A}, \mathcal{B}, \mathcal{C})$

- $\mathcal{X}$ is the set of variables $x_1, \ldots, x_{n_R}$ defined the same way in Γ,
- $\mathcal{A} = \{\mathcal{R}_1, \ldots, \mathcal{R}_{n_R}\}$
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We show that AC on Γ enforces BC(D) on any domain $\mathcal{D}$

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Bound(D) consistency is stronger than the classic bound consistency property

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- AC removes 5 from $\mathcal{D}(x_1), \mathcal{D}(x_2), \mathcal{D}(x_3), \mathcal{D}(x_4)$
- AC removes 5 from $\mathcal{D}(y_{1,1}), \mathcal{D}(y_{2,1}), \mathcal{D}(y_{3,1}), \mathcal{D}(y_{4,1})$
- No more propagation

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- No more propagation
- Assigning 3 to x_2 has no solution
- Γ hinders propagation!

Theorem

[Gusfield and Irving, 1989]

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- If a resident r_i is unassigned in one stable matching then it is unassigned in all stable matchings.
- If a hospital h_j is under-subscribed in one stable matching then it is assigned exactly the same residents in all stable matching

Preprocessing

- Compute the GS_lists and prune the domain accordingly ($O(L)$)

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- Compute the GS_lists and prune the domain accordingly (O(L))
- Notation: ***HFull*** the set of hospitals fully subscribed in any stable matching

Necessary and Sufficient Condition for Stability

Theorem

2-SidedStability($\mathcal{X}, \mathcal{A}, \mathcal{B}, \mathcal{C}$) is satisfiable iff

$$\forall\, 1 \leq j \leq n_H,\ \sum_{i=1}^{n_R} (\mathcal{R}_i[x_i] == j) \leq c_j \wedge$$

$$\forall\, 1 \leq i \leq n_R,\ \forall\, 1 \leq j \leq n_H + 1, x_i = j \implies$$

$$\forall k \in [1, j[$$

$$\textbf{if}\, h = \mathcal{R}_i[k] \textbf{then}$$

$$\sum_{m=1}^{n_R} (\mathcal{R}_m[x_m] == h) = c_h$$

$$\wedge$$

$$\forall l \underset{\mathcal{H}_h}{\succ\succ} i,\ \mathcal{R}_l[x_l] \neq h$$

Lemma

If Γ is AC then assigning all variables to their minimum value is solution.

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Theorem

Enforcing AC on Γ makes $\text{2-SidedStability}(\mathcal{X}, \mathcal{A}, \mathcal{B}, \mathcal{C})$ Bound($\mathcal{D}$) consistent.

Revisiting Bound(D) Consistency

- Best BC(D) algorithm runs in $O(c \times L)$ [Manlove et al., 2007]
- We propose an optimal algorithm running in $O(L)$

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Theorem

2-SidedStability($\mathcal{X}, \mathcal{A}, \mathcal{B}, \mathcal{C}$) is BC(D) iff assigning every variable to its maximum is a solution and assigning every variable to its minimum is a solution.

Lower bound changes

- Assume we have a domain that is BC(D)
- Let r_i be a resident whose lower bound has changed
- Let h_j be the hospital corresponding to the new lower bound

Step 1

- Let h be any hospital ‘between’ the old and the new lower bound
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$$\Uparrow$$

$$lastLeft_h$$

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Step 2

Make sure that the new hospital h_j has no more than c_j residents whose lower bound corresponds to h_j.

- MIN_{h_j}: The variables whose minimum corresponds to h_j
- If $|MIN_{h_j}| = c_{h_j} + 1$: Remove the worst resident r from MIN_{h_j} and prune h_j from the domain of r

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Example

$$\mathcal{H}_{h_j} \quad r_2 \quad r_8 \quad r_7 \quad r_{11} \quad r_9 \quad r_5 \quad r_6 \quad r_3 \quad r_1 \quad \underset{\substack{\Uparrow \\ lastLeft_h}}{r_8} \quad \cancel{r_6} \quad \cancel{r_{12}}$$

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Make sure that the new hospital h_j has no more than c_j residents whose lower bound corresponds to h_j.

- MIN_{h_j}: The variables whose minimum corresponds to h_j
- If $|MIN_{h_j}| = c_{h_j} + 1$: Remove the worst resident r from MIN_{h_j} and prune h_j from the domain of r

Example

$$\mathcal{H}_{h_j} \quad r_2 \quad r_8 \quad r_7 \quad r_{11} \quad r_9 \quad r_5 \quad r_6 \quad \cancel{r_3} \quad r_1 \quad r_8 \quad \cancel{r_6} \quad \cancel{r_{12}}$$

$$\Uparrow$$

$$lastLeft_h$$

Upper bound changes

- MAX_h: The indexes of residents where the maximum corresponds to h
- $maxofMAX_h = \max(MAX_h)$

Lemma

Assigning all variables to their maximum is a solution iff $\forall\ h \in \boldsymbol{HFull}$, $|MAX_h| = c_h$, and $\forall\ i \leq maxofMAX_h$, let $r = \mathcal{H}_h[i]$, and $l = \mathcal{R}_r{}^{-1}[h]$, then $i \notin MAX_h \implies \max(x_r) < l$.

Upper bound changes

- Assume we have a domain that is BC(D)
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- Not incremental

Beyond Constraint Propagation

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Hospital/Resident Problem with Forced and Forbidden Pairs

- Unknown complexity
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Stable Marriage Problem with Forced and Forbidden Pairs

- Best complexity $O(n^4)$ [Dias et al., 2003]
- Particular case of the above approach
- $O(n^2)$ to solve

Experiments

Problem description

- Some couples prefer to be matched together
- Same preferences for these couples
- Find a stable matching maximizing the number of such couples who are matched together

Experiments

Protocol

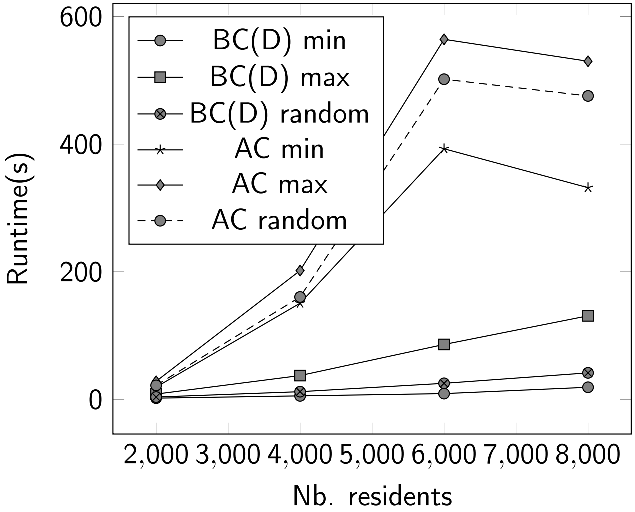
- Random instances: $2k, 4k, \ldots, 8k$ residents; $100, 200, \ldots 500$ hospitals; and different capacities $c \in \{100 + 50 + k | k \in [0, 8]\}$
- BC(D) and AC Implemented in Mistral-2.0
- LEX variable branching + (min/max/random min max) value branching
- Geometric restarts
- 5 different seeds for “random min max”
- 20 minutes time cutoff

Summary of the Results

Set	BC(D)-min			AC-min			BC(D)-max		
	Time	Nodes	Opt	Time	Nodes	Opt	Time	Nodes	Opt
2k	2	16.56	**100**	19	13.33	**100**	8	256.38	**100**
4k	5	18.14	**100**	151	14.69	**100**	37	394.86	**100**
6k	9	18.08	**100**	393	14.89	93	86	648.82	**100**
8k	19	18.16	**100**	332	15.80	79	131	491.50	**100**

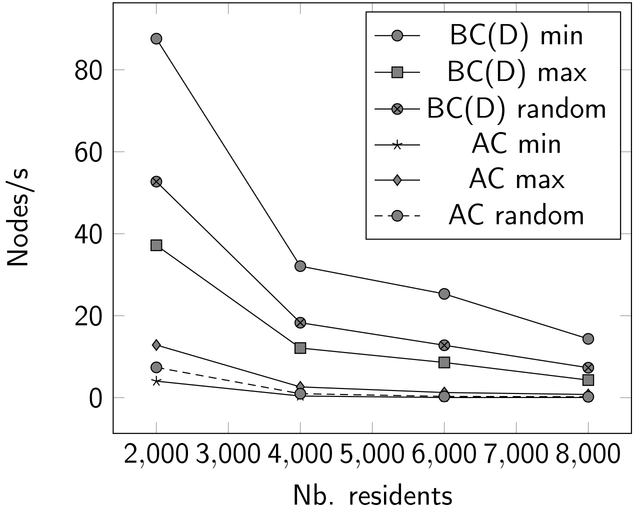
	AC-max			BC(D)-rand			AC-rand		
	Time	Nodes	Opt	Time	Nodes	Opt	Time	Nodes	Opt
2k	28	204.53	**100**	4	67.36	**100**	12	81.91	**100**
4k	202	320.33	**100**	12	81.91	**100**	160	65.61	**100**
6k	564	535.65	85	25	120.58	**100**	502	99.20	92
8k	530	432.87	69	42	98.88	**100**	475	88.11	77

Runtime



600
400
200
0
Runtime(s)
BC(D) min
BC(D) max
BC(D) random
AC min
AC max
AC random
2,000
3,000
4,000
5,000
6,000
7,000
8,000
Nb. residents

Speed of Exploration



Conclusion and Future Research

Contributions

- Previous CP propositions maintain only BC(D)
- A better implementation of BC(D) in $O(L)$ time
- An adaptation of BC(D) to achieve AC
- First polynomial algorithm to solve the Hospital/Resident problem with forced and forbidden pairs
- Improving the worst case complexity to solve SM with forced and forbidden pairs by a factor of $O(n^2)$

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Future Research

- Better implementation for AC?
- New global constraints are coming for different stable matching problems..

Thank you.

Picture taken from The New York Times

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