Search, propagation, and learning in sequencing and scheduling problems

Mohamed Siala

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Christian Bessiere	LIRMM Montpellier
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Emmanuel Hebrard	LAAS-CNRS Toulouse
George Katsirelos	INRA Toulouse
Christine Solnon	INSA Lyon

Context

Sequencing and Scheduling: the organization in time of of operations subject to capacity and resource constraints.

Mon 5	Tue 6	Wed 7	Thu 8	Fri 9	Sat 10	Sun 11	Mon 12	Tue 13	Wed 14	Thu 15	Fri 16	Sat 17	Sun 18
CD	EIA		EIA	LIK	NER		CD	EIA		EIA	LIK	NER	
LIK	LD		SS	EIF		NOO	LIK	SSN		SS	EIF		NOO
	CD	EIA	EIB	EIA	LIK	CD	EIA		EIA	LIK	NER		
	LIK	SSN		Conference L	EIF	LIK	SSN		Conference L	EIF		NOO	
CD	EIA												
LIK	EIA		EIA			NOO	CD	LIK		EIA	LIK	NER	
CD	EIA	LIK	EIA	LIK	NER		LIK		LIK	On Call	EIF		NOO
LIK	SSN		SS	EIF		NOO		CD					
		CD					CD	LIK	CD	EIA	LIK	LIK	
CD	EIA	LIK	EIA	LIK	NER		LIK	CD	LIK	Conference L	EIF		NOO
LIK	SSN	CD	Conference L	EIF			CD	EIA	CD	EIA	LIK	CD	
CD	EIA		EIA	LIK	NER		LIK	SSN		FL	EIF	LIK	NOO
LIK	LU		MLH									CD	
CD	EIA		EIA		NER		CD	EIA		EIA	LIK		
LIK	SSN		EIA				LIK	SSN		Conference L	EIF		NOO
	CD	EIA		EIA	LIK								
	LIK	SSN		Conference L	EIF		NOO						
CD	EIA		EIA	LIK	NER		CD	EIA		EIA	LIK	NER	
LIK	SSN		EIA	EIF		NOO	LIK	SSN		MLM	EIF		NOO
CD	EIA		EIG	LIK	NER		CD	EIA		EIA	LIK	NER	
LIK	LD		MLM	EIF		NOO	LIK	SSN		SST	EIF		NOO

PhD Context

- Combinatorial (optimization) problems
- Constraint satisfaction and optimization

- Laboratory: LAAS-CNRS, Toulouse
- Research Team: ROC
- Supervision: Christian Artigues, and Emmanuel Hebrard
- Funding:

Thesis overview

Constraint Programming: Search $\oplus$ Propagation

Thesis overview

Constraint Programming: Search $\oplus$ Propagation $\oplus$ **Learning**

Thesis overview

Constraint Programming: Search $\oplus$ Propagation $\oplus$ **Learning**

All these aspects are important and must all be taken into account in order to design efficient solution methods

Outline

Definition

A constraint is a finite relation

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Definition

A `constraint network` (CN) is defined by a triplet $\mathcal{P} = (\mathcal{X}, \mathcal{D}, \mathcal{C})$ where

- $\mathcal{X} = [x_1, \ldots, x_n]$: finite set of variables
- $\mathcal{D}$: a domain for $\mathcal{X}$
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- CSP is NP-Hard in general

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- Complete backtracking algorithms

Search

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Variable Ordering

'Fail-first' principle [Haralick and Elliott, 1980]:

"To succeed, try first where you are most likely to fail"

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Variable Ordering

'Fail-first' principle [Haralick and Elliott, 1980]:

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Value Ordering

'succeed-first' [Geelen, 1992]:

Best chances leading to a solution

Propagation

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- Propagation: inferences based on the current state
- Constraint $\leftrightarrow$ a propagator
- Propagators are executed sequentially before taking any decision
- The level of pruning $\leftrightarrow$ local consistency

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Arc Consistency

- Let $\mathcal{D}$ be a domain, and C be a constraint
- C is Arc Consistent (AC) iff for every x in the scope of C, for every value $v \in \mathcal{D}(x)$ there exists an assignment w in $\mathcal{D}$ satisfying C in which v is assigned to x

Learning

Learning

$x_1 + x_7 \geq 4 \wedge$
$x_2 + x_{10} \geq 11 \wedge$
$x_3 + x_9 = 16 \wedge$
$x_5 \geq x_8 + x_9 \wedge$
$b \leftrightarrow (x_9 - x_4 = 14) \wedge$
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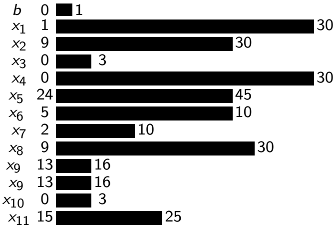

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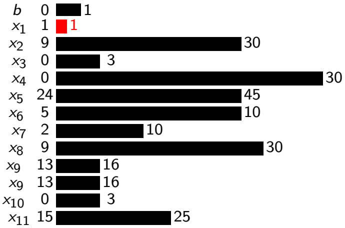
$[\![x_1 = 1]\!]$



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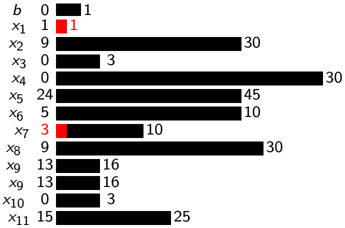

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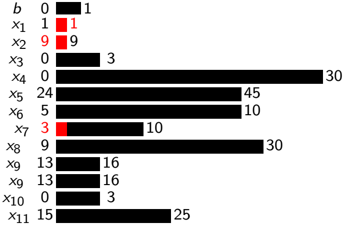

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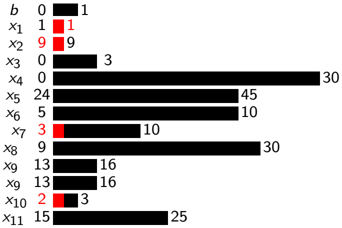

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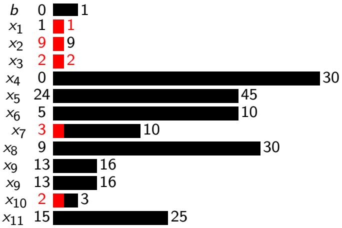

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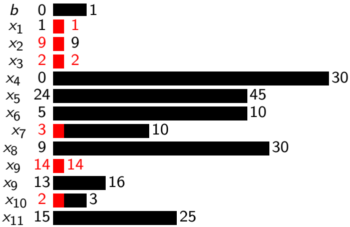

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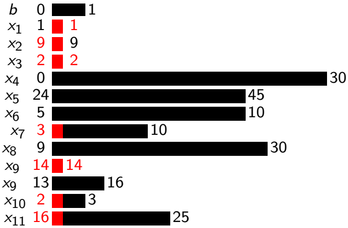

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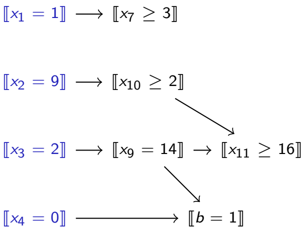

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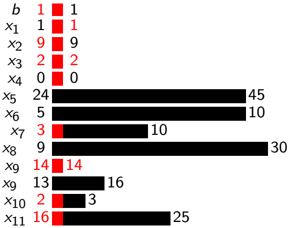

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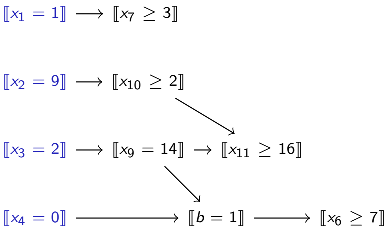

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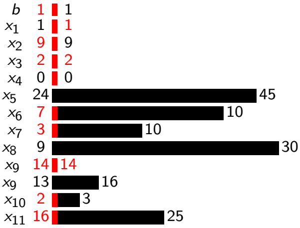

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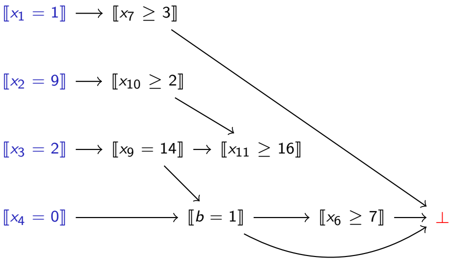

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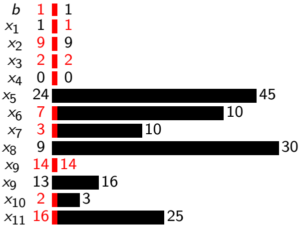

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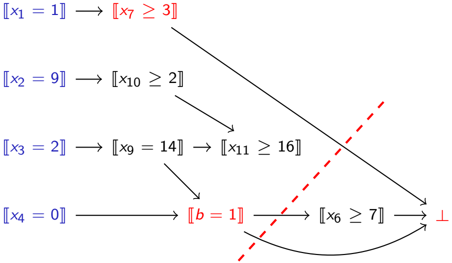

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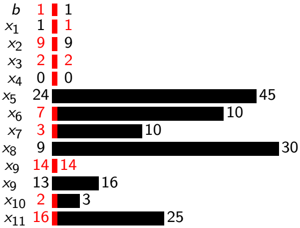

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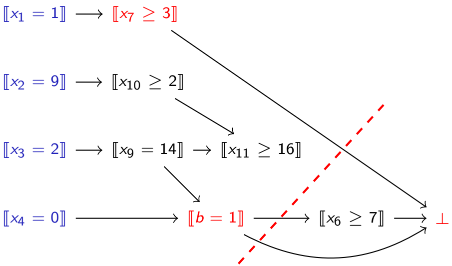

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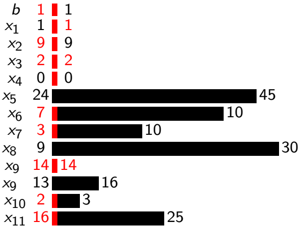

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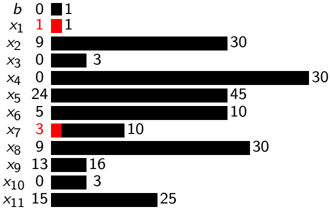

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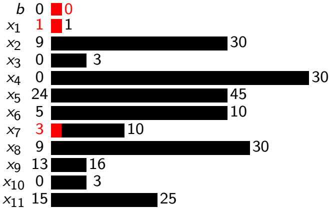
- Propagate the learnt clause



Learning

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- New clause: $[\![b \neq 0]\!] \vee [\![x_7 \leq 2]\!]$
- Backtrack to level 1
- Propagate the learnt clause
- Continue exploration

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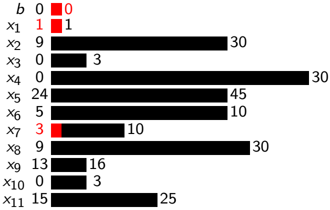

Learning in CP

- Hybrid CP/SAT
- Based on the notion of explanation
- Conflict Driven Clause Learning (CDCL) [Moskewicz et al., 2001]

Summary of the thesis

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Modern CP-Solvers may not underestimate any of the three aspects: search, propagation, and learning

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- Search in car-sequencing

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- Search in car-sequencing
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- Search in car-sequencing
- Propagation in a class of sequencing problems
- Learning in car-sequencing
- Revisiting lazy generation

Summary of the thesis

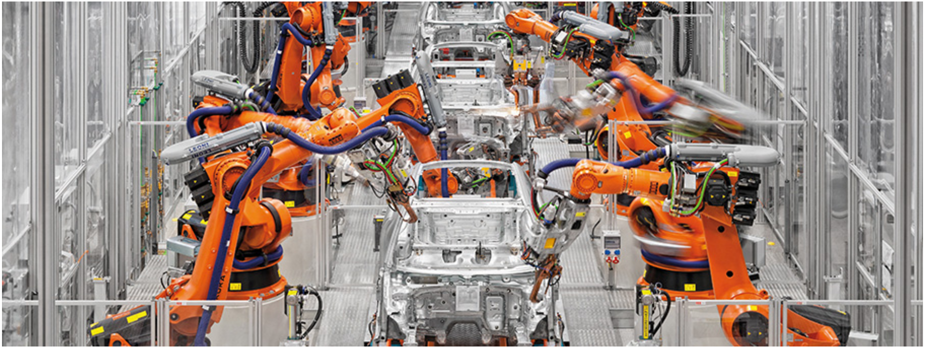
Modern CP-Solvers may not underestimate any of the three aspects: search, propagation, and learning

Contributions

- Search in car-sequencing
- Propagation in a class of sequencing problems
- Learning in car-sequencing
- Revisiting lazy generation
- Learning in disjunctive scheduling

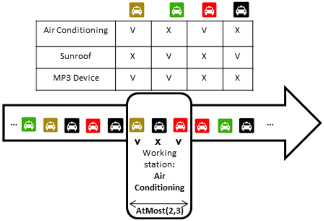
Outline

Car-Sequencing



- ROADEF'05 challenge [Solnon et al., 2008]
- RENAULT

Problem Definition



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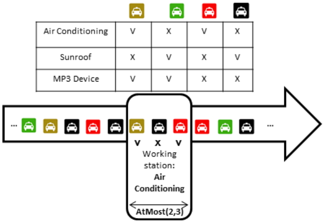

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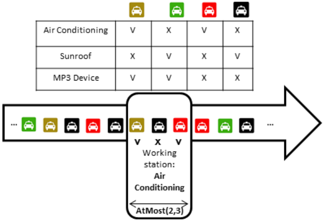

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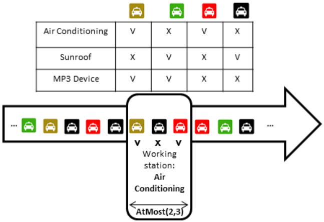

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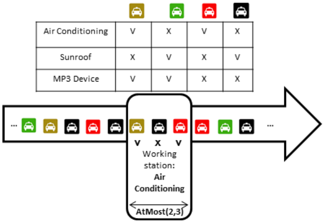

- A class of vehicles is defined by a set of options
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- `Capacity` constraints: no subsequence of size q may contain more than p vehicles requiring a given option
- Is there a sequence of cars satisfying both demand and capacity constraints?

Outline

Propagation via AtMostSeqCard

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Definition

$\textsc{AtMostSeqCard}(p, q, d, [x_1, \ldots, x_n]) \Leftrightarrow$

$$\bigwedge_{i=0}^{n-q} (\sum_{l=1}^{q} x_{i+l} \leq p) \wedge (\sum_{i=1}^{n} x_i = d)$$

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Example $\textsc{AtMostSeqCard}(2, 5, 4, [x_1, \ldots, x_9])$

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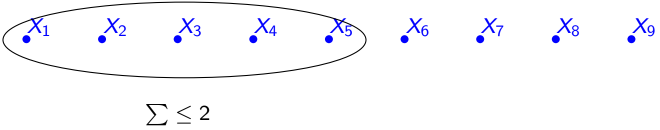

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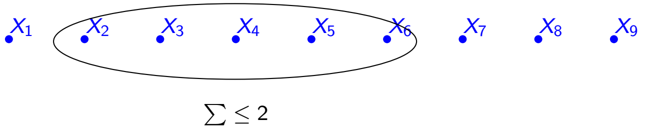

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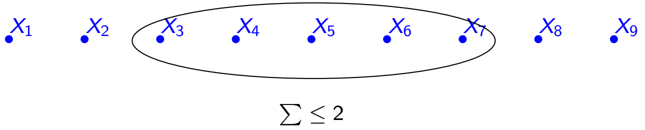

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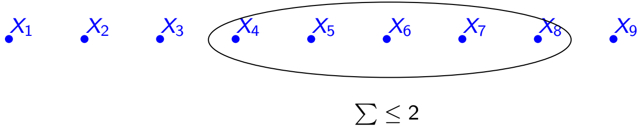

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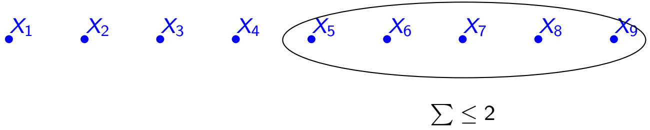

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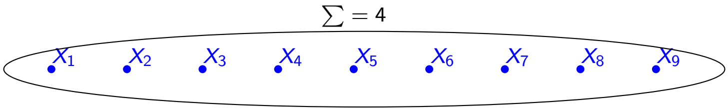

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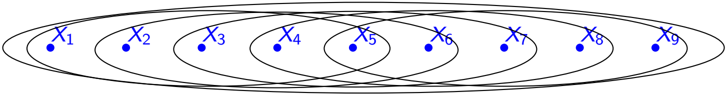
$\sum = 4$



$\sum \leq 2$ $\sum \leq 2$ $\sum \leq 2$ $\sum \leq 2$ $\sum \leq 2$

AtMostSeqCard as a global constraint?

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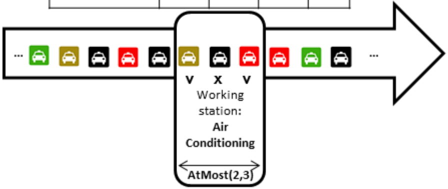
1. Car sequencing

Air Conditioning	V	X	V	X
Sunroof	X	V	X	V
MP3 Device	V	V	X	X



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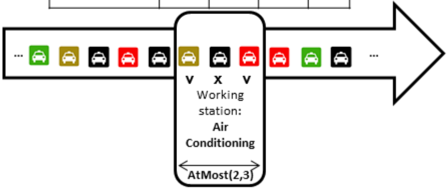

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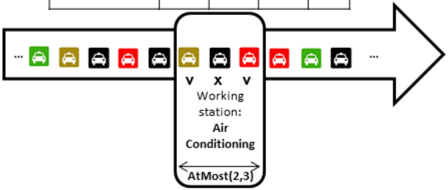

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Arc Consistency

$$\textsc{AtMostSeqCard}(p, q, d, [x_1, \ldots, x_n]) \Leftrightarrow$$

$$\textsc{AtMostSeq}(p, q, [x_1, \ldots, x_n]) \wedge \textsc{Cardinality}(d, [x_1, \ldots, x_n])$$

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$\textsc{AtMostSeqCard}$ as a particular case?

- $\textsc{cost-Regular}$: $O(2^q n)$ [van Hoeve et al., 2009]
- $\textsc{Gen-Sequence}$: $O(n^3)$ [van Hoeve et al., 2009]
- $\textsc{Gen-Sequence}$: $O(n^2.log(n))$ down a branch $\oplus$ initial compilation of $O(q.n^2)$. [Maher et al., 2008].

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Key idea

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- Complete the filtering based on a greedy rule

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Maximum possible 9 < residual demand

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Achieving Arc consistency

- `leftmost`: a greedy rule computing an assignment w of maximum cardinality with respect to AtMostSeq.
- `leftmost_count`: a linear implementation returning for each i the maximum cardinality that can be added until i
- L: `leftmost_count` from left to right
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Achieving AC in linear time

1. AC on ATMOSTSEQ and CARDINALITY
2. - If $L[n] < d_{res}$: failure
   - If $L[n] = d_{res}$, then $\forall i$:
     - If $L[i] + R[n-i+1] \leq d_{res}$, then x_i is assigned to 0.
     - If $L[i-1] + R[n-i] < d_{res}$, then x_i is assigned to 1.

Example

$\textsc{AtMostSeqCard}(4, 8, 12)$

. 0 . . . . . . 0 1 0 1

Example

AtMostSeqCard$(4, 8, 12)$

	.	0	.	.	.	.	.	.	0	1	0	.	.	.	.	.	.	.	.	.	.	1
$\overrightarrow{w}[i]$	**1**	0	**1**	**1**	**1**	0	0	0	0	1	0	**1**	**1**	**1**	0	0	0	**1**	0	**1**	**1**	1

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$\textsc{AtMostSeqCard}(4, 8, 12)$

		.	0	.	.	.	.	.	.	0	1	0	.	.	.	.	.	.	.	.	.	.	1
$\overrightarrow{w}[i]$		**1**	0	**1**	**1**	**1**	0	0	0	0	1	0	**1**	**1**	**1**	0	0	0	**1**	0	**1**	**1**	1
$L[i]$	0	1	1	2	3	4	4	4	4	4	4	4	5	6	7	7	7	7	8	8	9	10	10

Example

AtMostSeqCard$(4, 8, 12)$

		.	0	.	.	.	.	.	.	0	1	0	.	.	.	.	.	.	.	.	.	.	1
$\overrightarrow{w}[i]$		**1**	0	**1**	**1**	**1**	0	0	0	0	1	0	**1**	**1**	**1**	0	0	0	**1**	0	**1**	**1**	1
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$\overleftarrow{w}[i]$		**1**	0	0	**1**	**1**	**1**	0	0	0	1	0	**1**	**1**	**1**	0	0	0	0	**1**	**1**	**1**	1

Example

AtMostSeqCard(4, 8, 12)

		.	0	.	.	.	.	.	.	0	1	0	.	.	.	.	.	.	.	.	.	.	1	
$\overrightarrow{w}[i]$		**1**	0	**1**	**1**	**1**	0	0	0	0	1	0	**1**	**1**	**1**	0	0	0	**1**	0	**1**	**1**	1	
$L[i]$	0	1	1	2	3	4	4	4	4	4	4	4	5	6	7	7	7	7	8	8	9	10	10	
$\overleftarrow{w}[i]$		**1**	0	0	**1**	**1**	**1**	0	0	0	1	0	**1**	**1**	**1**	0	0	0	0	**1**	**1**	**1**	1	
$R[i]$		10	9	9	9	8	7	6	6	6	6	6	6	5	4	3	3	3	3	3	2	1	0	0

Example

$\textsc{AtMostSeqCard}(4, 8, 12)$

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	
		.	0	.	.	.	.	.	.	0	1	0	.	.	.	.	.	.	.	.	.	.	1	
$\overrightarrow{w}[i]$		**1**	0	**1**	**1**	**1**	0	0	0	0	1	0	**1**	**1**	**1**	0	0	0	**1**	0	**1**	**1**	1	
$L[i]$	0	1	1	2	3	4	4	4	4	4	4	4	5	6	7	7	7	7	8	8	9	10	10	
$\overleftarrow{w}[i]$		**1**	0	0	**1**	**1**	**1**	0	0	0	1	0	**1**	**1**	**1**	0	0	0	0	**1**	**1**	**1**	1	
$R[i]$		10	9	9	9	8	7	6	6	6	6	6	6	5	4	3	3	3	3	3	2	1	0	0
$L[i] + R[n-i+1]$		11	10	11	12	12	11	10	10	10	10	10	11	11	11	10	10	10	11	11	11	11	10	
$L[i-1] + R[n-i]$		9	10	10	10	10	10	10	10	10	10	10	9	9	9	10	10	10	10	10	9	9	10	

Example

$\textsc{AtMostSeqCard}(4, 8, 12)$

		.	0	.	.	.	.	.	.	0	1	0	.	.	.	.	.	.	.	.	.	.	1	
$\overrightarrow{w}[i]$		**1**	0	**1**	**1**	**1**	0	0	0	0	1	0	**1**	**1**	**1**	0	0	0	**1**	0	**1**	**1**	1	
$L[i]$	0	1	1	2	3	4	4	4	4	4	4	4	5	6	7	7	7	7	8	8	9	10	10	
$\overleftarrow{w}[i]$		**1**	0	0	**1**	**1**	**1**	0	0	0	1	0	**1**	**1**	**1**	0	0	0	0	**1**	**1**	**1**	1	
$R[i]$		10	9	9	9	8	7	6	6	6	6	6	6	5	4	3	3	3	3	3	2	1	0	0
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$L[i-1] + R[n-i]$		9	10	10	10	10	10	10	10	10	10	10	9	9	9	10	10	10	10	10	9	9	10	
AC		**1**	0	.	.	.	.	**0**	**0**	0	1	0	**1**	**1**	**1**	**0**	**0**	**0**	.	.	**1**	**1**	1	

Experimental Results

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Variables

- Class variables: n integer variables $\{x_1, \ldots, x_n\}$
- Option variables: nm Boolean variables $\{y_1^1, \ldots, y_n^m\}$

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Constraints

1. *Demand constraints*: $\forall c \in \{1..k\}, \left|\{i \mid x_i = c\}\right| = d_c^{class}$: Global Cardinality Constraint.
2. *Capacity constraints*:
   1. A naive decomposition: DECOMPOSITION
   2. Global Sequencing Constraint: GSC [Régin and Puget, 1997]
   3. ATMOSTSEQCARD: AMSC
   4. Combine ATMOSTSEQCARD and GSC: GSC⊕AMSC
3. *Channeling*: between option and class variables

Experimental results: Car-Sequencing

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	set1 (70 × 42 × 5)				set2 (4 × 42 × 5)			
	#sol	avg bts	max bts	time	#sol	avg bts	max bts	time
DECOMPOSITION	8480	231.2K	25.0M	13.93	95	1.4M	15.3M	76.60
GSC	11218	1.7K	2.3M	3.60	325	131.7K	1.5M	110.99
ATMOSTSEQCARD	10702	39.1K	13.8M	4.43	360	690.8K	10.2M	72.00
GSC⊕AMSC	11243	1.2K	1.1M	3.43	339	118.4K	1.0M	106.53

	set3 (5 × 42 × 5)				set4 (7 × 42 × 5)			
	#sol	avg bts	max bts	time	#sol	avg bts	max bts	time
DECOMPOSITION	0	-	-	$>$ 1200	64	543.3K	13.7M	43.81
GSC	31	55.3K	218.5K	276.06	140	25.2K	321.9K	56.61
ATMOSTSEQCARD	16	40.3K	83.4K	8.62	153	201.4K	3.2M	33.56
GSC⊕AMSC	32	57.7K	244.7K	285.43	147	23.8K	371.0K	66.45

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	set1 (70 × 42 × 5)				set2 (4 × 42 × 5)			
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	#sol	avg bts	max bts	time	#sol	avg bts	max bts	time
DECOMPOSITION	0	-	-	> 1200	64	543.3K	13.7M	43.81
GSC	31	55.3K	218.5K	276.06	140	25.2K	321.9K	56.61
ATMOSTSEQCARD	16	40.3K	83.4K	8.62	153	201.4K	3.2M	33.56
GSC⊕AMSC	32	57.7K	244.7K	285.43	147	23.8K	371.0K	66.45

- Best Models: ATMOSTSEQCARD and ATMOSTSEQCARD ⊕ GSC
- GSC saves more backtracks than ATMOSTSEQCARD but extremely slow

Experimental results: Car-Sequencing

	set1 (70 × 42 × 5)				set2 (4 × 42 × 5)			
	#sol	avg bts	max bts	time	#sol	avg bts	max bts	time
DECOMPOSITION	8480	231.2K	25.0M	13.93	95	1.4M	15.3M	76.60
GSC	11218	1.7K	2.3M	3.60	325	131.7K	1.5M	110.99
ATMOSTSEQCARD	10702	39.1K	13.8M	4.43	360	690.8K	10.2M	72.00
GSC⊕AMSC	11243	1.2K	1.1M	3.43	339	118.4K	1.0M	106.53

	set3 (5 × 42 × 5)				set4 (7 × 42 × 5)			
	#sol	avg bts	max bts	time	#sol	avg bts	max bts	time
DECOMPOSITION	0	-	-	> 1200	64	543.3K	13.7M	43.81
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- Best Models: ATMOSTSEQCARD and ATMOSTSEQCARD ⊕ GSC
- GSC saves more backtracks than ATMOSTSEQCARD but extremely slow
- [van Hoeve et al., 2009] 65.2% while GSC⊕AMSC 96.20%

Extensions for AtMostSeqCard

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$$\textsc{MultiAtMostSeqCard}(p_1, .., p_m, q_1, .., q_m, d, [x_1, \dots, x_n]) \Leftrightarrow$$

$$\bigwedge_{k=1}^{m} \bigwedge_{i=0}^{n-q_k} (\sum_{l=1}^{q_k} x_{i+l} \leq p_k) \wedge (\sum_{i=1}^{n} x_i = d)$$

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- The decomposition into m AtMostSeqCard is hindering propagation
- The filtering for AtMostSeqCard can be adapted to achieve AC in $O(m \times n)$
- MultiAtMostSeqCard outperforms the other models in crew-rostering

Publications

- [Honorable mention] *An optimal arc consistency algorithm for a chain of atmost constraints with cardinality*
  Mohamed Siala, Emmanuel Hebrard, and Marie-José Huguet. In *Principles and Practice of Constraint Programming - 18th International Conference, CP 2012, Québec City, QC, Canada, October 8-12, 2012*
- *An optimal arc consistency algorithm for a particular case of sequence constraint*
  Mohamed Siala, Emmanuel Hebrard, and Marie-José Huguet. *Constraints*, 19(1):30–56, 2014

Outline

Related work regarding the search strategy

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Motivation

Can we combine these heuristics in one structure?

New Classification

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- `Branching`: *class*, *option*.

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- `Selection`:
  1. capacity p_j/q_j
  2. demand d_j^{opt}
  3. load $\delta_j = d_j^{opt}.\frac{q_j}{p_j}$
  4. slack $\sigma_j = n - (n_j - \delta_j)$
  5. usage rate $\rho_j = \delta_j/n_j$

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Overall 42 heuristics

$\langle\{class, option\}, \{lex, middle\}, \{q/p, d^{opt}, \delta, n{-}\sigma, \rho, 1\}, \{\leq_{\sum}, \leq_{Euc}, \leq_{lex}\}\rangle$

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Summary

- Many good heuristics raise as untested combinations
- Branching and Selection are the most crucial criteria
- The most robust heuristics:
  $\langle class, \{lex, middle\}, \delta, \{\leq_{\sum}, \leq_{Euc}, \leq_{lex}\}\rangle$
- Search is as important as propagation based on two metrics *confidence* and *significance*

Publication

A study of constraint programming heuristics for the car-sequencing problem.
Mohamed Siala, Emmanuel Hebrard, and Marie-José Huguet. *Engineering Applications of Artificial Intelligence*, 38(0):34 – 44, 2015

Outline

1 Context

2 Background

3 Case Study: The Car-Sequencing Problem
- Propagation
- Search
- Learning

4 Learning in Disjunctive Scheduling

5 Conclusions & Perspectives

Hybrid CP/SAT Models

- Models based on AtMostSeqCard

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Explaining AtMostSeqCard?

- Explain AtMostSeq and Cardinality
- Explaining the extra filtering: consider the naive explanation, then try to reduce it.

Explaining failure: key idea

- `leftmost`: a greedy rule computing an assignment w of maximum cardinality with respect to AtMostSeq.
- max: a vector containing for each i the maximum cardinality in w of all subsequences involving i

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Observations

Let S: 1 1 0 0 . subject to AtMost(2/5)

→`leftmost` on S gives 1 1 0 0 0

Consider the sequence S_0: 1 1 . 0 .

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`leftmost` on S_2 gives 1 1 0 0 0

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Consider the sequence S_2: . 1 0 0 .

`leftmost` on S_2 gives 1 1 0 0 0

Always true when $\{[\![x_i = 1]\!] \mid max(i) \neq p\}$

Reduced Explanations

Reduced Explanations

A weaker domain $\widehat{\mathcal{D}}$ defined as follows:

$$
\begin{array}{ll}
\widehat{\mathcal{D}}(x_i) = \{0,1\} & \text{if } \mathcal{D}(x_i) = \{0\} \wedge max(i) = p \\
\widehat{\mathcal{D}}(x_i) = \{0,1\} & \text{if } \mathcal{D}(x_i) = \{1\} \wedge max(i) \neq p \\
\widehat{\mathcal{D}}(x_i) = \mathcal{D}(x_i) & \text{otherwise}
\end{array}
$$

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Theorem

If a failure is raised because $L[n] < d_{res}$, then the set of assignments in $\widehat{\mathcal{D}}$ is a valid nogood.

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Theorem

If a failure is raised because $L[n] < d_{res}$ *, then the set of assignments in* $\widehat{\mathcal{D}}$ *is a valid nogood.*

Time Complexity

$O(n)$ since we call `leftmost_count` once to built *max*

Example

$\mathcal{D}$: 1 1 0 0 0 0 0 0 1 1 0 0 0 1 0 0 0 0 1 0 0 . . . 1

Example

$\mathcal{D}$: 1 1 0 0 0 0 0 0 1 1 0 0 0 1 0 0 0 0 1 0 0 . . . 1

Example

$\mathcal{D}$:	1	1	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	1	0	0	.	.	.	1
w	1	1	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	1

Extra filtering → Failure

Example

$\mathcal{D}$:	1	1	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	1	0	0	.	.	.	1
w	1	1	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	1

Extra filtering $\rightarrow$ Failure

max	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	2	2	2

Example

$\mathcal{D}$:	1	1	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	1	0	0	.	.	.	1
w	1	1	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	1

Extra filtering $\rightarrow$ Failure

max	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	2	2	2
$\widehat{\mathcal{D}}$:	1	1	.	.	.	.	.	.	1	1	.	.	.	.	0	0	0	0	.	0	0	.	.	.	1

Example

$\mathcal{D}$:	1	1	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	1	0	0	.	.	.	1
w	1	1	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	1

Extra filtering → Failure

max	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	2	2	2
$\widehat{\mathcal{D}}$:	1	1	.	.	.	.	.	.	1	1	.	.	.	.	0	0	0	0	.	0	0	.	.	.	1

Example

$\mathcal{D}$:	1	1	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	1	0	0	.	.	.	1
w	1	1	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	1

Extra filtering $\rightarrow$ Failure

max	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	2	2	2
$\widehat{\mathcal{D}}$:	1	1	.	.	.	.	.	.	1	1	.	.	.	.	0	0	0	0	.	0	0	.	.	.	1
w	1	1	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	1

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w	1	1	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	1

Extra filtering $\rightarrow$ Failure

max	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	2	2	2
$\widehat{\mathcal{D}}$:	1	1	.	.	.	.	.	.	1	1	.	.	.	.	0	0	0	0	.	0	0	.	.	.	1
w	1	1	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	1

Extra filtering $\rightarrow$ Failure

Size: 22 with naive explanation and 11 with reduced explanation

Example

$\mathcal{D}$:	1	1	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	1	0	0	.	.	.	1
w	1	1	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	1	0	0	1	0	0	1

Extra filtering $\rightarrow$ Failure

max	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1	2	2	2
$\widehat{\mathcal{D}}$:	1	1	.	.	.	.	.	.	1	1	.	.	.	.	0	0	0	0	.	0	0	.	.	.	1
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Note: not minimal

Experimental Results

Method	sat[easy] (74 × 5)			sat[hard] (7 × 5)			unsat* (28 × 5)		
	#suc	avg fails	time	#suc	avg fails	time	#suc	avg fails	time
CNF_A	370	2073	1.71	28	337194	282.35	**85**	**249301**	**105.07**
CNF_S	370	1114	0.87	31	60956	56.49	65	220658	197.03
CNF_{A+S}	370	612	0.91	34	32711	36.52	77	190915	128.09
hybrid (VSIDS)	370	903	0.23	16	207211	286.32	35	177806	224.78
hybrid (VSIDS/Slot)	370	739	0.23	35	76256	64.52	37	204858	248.24
hybrid (Slot/VSIDS)	370	132	0.04	34	4568	2.50	37	234800	287.61
hybrid (Slot)	370	132	0.04	**35**	**6304**	**3.75**	23	174097	299.24
CP	**370**	**43**	**0.03**	35	57966	16.25	0	-	-
PBO-clauses	277	538743	236.94	0	-	-	43	175990	106.92
PBO-cutting planes	272	2149	52.62	0	-	-	1	5031	53.38

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- Finding solutions quickly: Propagation is very important to find solutions quickly when they exist.
- For proving unsatisfiability: Clause learning is by far the most critical factor.

Publication

SAT and Hybrid Models of the Car-Sequencing problem
Christian Artigues, Emmanuel Hebrard, Valentin Mayer-Eichberger, Mohamed Siala, and Toby Walsh. In *Integration of AI and OR Techniques in Constraint Programming - 11th International Conference, CPAIOR 2014, Cork, Ireland, May 19-23, 2014*

Outline

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Disjunctive Scheduling

A family of scheduling problems having in common the `Unary Resource Constraint`.

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- Decomposition using the following DISJUNCTIVE constraints:

$$\delta_{kij} = \begin{cases} 0 & \Leftrightarrow \quad t_{ik} + p_{ik} \leq t_{jk} \\ 1 & \Leftrightarrow \quad t_{jk} + p_{jk} \leq t_{ik} \end{cases} \tag{1}$$

Context

Disjunctive Scheduling

A family of scheduling problems having in common the `Unary Resource Constraint`.

Unary Resource Constraint [Grimes and Hebrard, 2015]

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Our Contributions

- Alternative lazy generation approach
- Novel conflict analysis scheme

Revisiting Lazy Generation

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Standard Lazy Encoding

- Generate atoms lazily when learning new clauses.
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- There is a redundancy issue

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Atoms	clauses
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Revisiting Lazy Generation

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Example

Atoms	clauses
$\emptyset$	$\emptyset$
$[\![x \leq 57]\!]$	$\emptyset$
$[\![x \leq 317]\!]$	$[\![x \geq 57]\!] \vee [\![x \leq 317]\!]$

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$[\![x \leq 203]\!]$	$([\![x \geq 203]\!] \vee [\![x \leq 317]\!]) \wedge ([\![x \geq 57]\!] \vee [\![x \leq 203]\!])$

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$[\![x \leq 203]\!]$	$([\![x \geq 203]\!] \vee [\![x \leq 317]\!]) \wedge ([\![x \geq 57]\!] \vee [\![x \leq 203]\!])$
$[\![x \leq 84]\!]$	$([\![x \geq 84]\!] \vee [\![x \leq 203]\!]) \wedge ([\![x \geq 57]\!] \vee [\![x \leq 84]\!])$

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$[\![x \leq 203]\!]$	$([\![x \geq 203]\!] \vee [\![x \leq 317]\!]) \wedge ([\![x \geq 57]\!] \vee [\![x \leq 203]\!])$
$[\![x \leq 84]\!]$	$([\![x \geq 84]\!] \vee [\![x \leq 203]\!]) \wedge ([\![x \geq 57]\!] \vee [\![x \leq 84]\!])$
$[\![x \leq 250]\!]$	$([\![x \geq 250]\!] \vee [\![x \leq 317]\!]) \wedge ([\![x \geq 203]\!] \vee [\![x \leq 250]\!])$

Revisiting Lazy Generation

Standard Lazy Encoding

- Generate atoms lazily when learning new clauses.
- Generate related domain clauses.
- There is a redundancy issue

Example

Atoms	clauses
$\emptyset$	$\emptyset$
$[\![x \leq 57]\!]$	$\emptyset$
$[\![x \leq 317]\!]$	$[\![x \geq 57]\!] \vee [\![x \leq 317]\!]$
$[\![x \leq 203]\!]$	$([\![x \geq 203]\!] \vee [\![x \leq 317]\!]) \wedge ([\![x \geq 57]\!] \vee [\![x \leq 203]\!])$
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$O(k)$ redundant clauses

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Arc consistency

Can be enforced in constant amortized time complexity ($O(1)$) down a branch of the search tree

DISJUNCTIVE-based Learning

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Disjunctive-based Learning

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1. First UIP cut with a reified Boolean variable
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- $\oplus$ No domain encoding
- $\oplus$ Scheduling horizon does not manner in size
- $\ominus$ Language of literals is restricted compared to standard approaches

Experimental results

Protocol

- **Mistral-Hybrid:** new hybrid solver with
  - backward explanation
  - semantic reductions
  - lazy generation
  - DISJUNCTIVE-based learning
- **https://github.com/siala/Hybrid-Mistral**
- Job Shop and Open Shop benchmarks

Experimental results: Job Shop

Experimental results: Job Shop

Lawrence results

Mistral(*task*)		Hybrid(*vsids*, *disj*)		Hybrid(*vsids*, *lazy*)		Hybrid(*task*, *disj*)		Hybrid(*task*, *disj*)	
T	%O	T	%O	T	%O	T	%O	T	%O
471.97	88.75	**396.20**	**92**	602.51	88	410.55	90.50	489.25	89

Taillard results

	Mistral(*task*)			Hybrid(*vsids*, *disj*)			Hybrid(*vsids*, *lazy*)			Hybrid(*task*, *disj*)			Hybrid(*task*, *lazy*)		
	M		Size	M		Nodes/S	M		Nodes/S	M		Nodes/S	M		Nodes/S
	%O	T		%O	T		%O	T		%O	T		%O	T	
t01-10	90	616.22	8871.32	**90**	**477.79**	6814.73	87	999.17	1213.57	90	574.87	4869.45	85	1115.49	1261.70
	PRD			PRD			PRD			PRD			PRD		
t11-20	3.2381		6509.44	3.0350		3970.85	1.8937		520.62	0.4808		2715.29	**0.1169**		539.79
t21-30	0.7302		3935.87	0.2769		2424.16	0.4756		413.90	0.2485		1752.05	**0.1557**		437.04
t31-40	1.7227		4503.78	0.7109		2598.25	**0.3043**		555.36	0.6016		1517.04	0.4103		566.18
t41-50	2.2161		2570.10	0.4798		1530.42	**0.3036**		413.48	0.5420		994.61	0.6163		443.63
t51-60	2.0798		1952.51	2.2847		2602.31	2.7990		562.71	**0.1621**		1131	0.2419		698.37
t61-70	3.2381		1349.73	3.0350		2183.79	1.8937		522.25	0.4808		920.55	**0.1169**		584.14

- PRD: percentage relative deviation

Experimental results: Summary

- 'Light' CP models are extremely efficient with small sized instances
- These models benefit essentially from the fast exploration speed
- The impact of clause learning is more and more glaring when the size of the instance grows
- DISJUNCTIVE-based learning outperforms the other models on medium sized instances

Experimental results: lower bounds experiments

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Open instances from Taillard benchmark

- 7 new bounds found with DISJUNCTIVE-based and VSIDS

tai13		tai21		tai23		tai25		tai26		tai29		tai30	
new	old	new	old	new	old	new	old	new	old	new	old	new	old
1305	1282	1613	1573	1514	1474	1543	1518	1561	1558	1573	1525	1508	1485

Experimental results: lower bounds experiments

Open instances from Taillard benchmark

- 7 new bounds found with DISJUNCTIVE-based and VSIDS

tai13		tai21		tai23		tai25		tai26		tai29		tai30	
new	old	new	old	new	old	new	old	new	old	new	old	new	old
1305	1282	1613	1573	1514	1474	1543	1518	1561	1558	1573	1525	1508	1485
1342		**1642**		**1518**		**1558**		**1591**		**1573**		**1519**	

[Vilím et al., 2015]

- IBM CP-Optimizer studio
- 8h20min per instance
- Parallelization: Double threading phase
- Start search with best known bounds as an additional information.

Outline

Summary

- Contributions to each of the three aspects of constraint programming that are ‘search’, ‘propagation’ and ‘learning’ for efficiently solving sequencing and scheduling problems.

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- Case study: car-sequencing
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Modern constraint programming solvers may not underestimate any of these three aspects

Future Research

- Car-Sequencing:
  - Application to 'real' industrial situations [Solnon et al., 2008].
- Propagation via AtMostSeqCard:
  - Incrementality?
  - More extensions?
- Explanation for AtMostSeqCard:
  - Minimal explanations?
  - Applications to other sequencing problems.
- Learning in Scheduling Problems:
  - Applications to other scheduling problems.
  - Learning with global constraints.
  - Hand-crafted learning.

Thank you.

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