Search, propagation, and learning in sequencing and scheduling problems

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Insight
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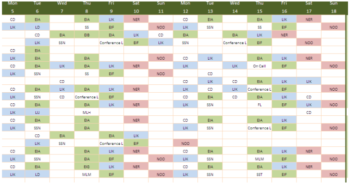
UCC Coláiste na hOllscoile Corcaigh, Éire University College Cork, Ireland

Context

Sequencing and Scheduling: the organization in time of of operations subject to capacity and resource constraints.



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	LIK	SSN		Conference L	EIF	LIK	SSN		Conference L	EIF		NOO	
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LIK	SSN		EIA				LIK	SSN		Conference L	EIF		NOO
	CD	EIA		EIA	LIK								
	LIK	SSN		Conference L	EIF		NOO						
CD	EIA		EIA	LIK	NER		CD	EIA		EIA	LIK	NER	
LIK	SSN		EIA	EIF		NOO	LIK	SSN		MLM	EIF		NOO
CD	EIA		EIG	LIK	NER		CD	EIA		EIA	LIK	NER	
LIK	LD		MLM	EIF		NOO	LIK	SSN		SST	EIF		NOO

PhD Context

- Combinatorial (optimization) problems
- Constraint satisfaction and optimization

- Laboratory: LAAS-CNRS, Toulouse
- Research Team: ROC (Operations Research, Combinatorial Optimization and Constraints)
- Supervision: Christian Artigues, and Emmanuel Hebrard
- Funding:

Thesis overview

Constraint Programming: Search $\oplus$ Propagation

Thesis overview

Constraint Programming: Search $\oplus$ Propagation $\oplus$ **Learning**

Thesis overview

Constraint Programming: Search $\oplus$ Propagation $\oplus$ **Learning**

All these aspects are important and must all be taken into account in order to design efficient solution methods

Outline

Definition

A constraint is a finite relation

Definition

A constraint is a finite relation

Definition

A `constraint network` (CN) is defined by a triplet $\mathcal{P} = (\mathcal{X}, \mathcal{D}, \mathcal{C})$ where

- $\mathcal{X} = [x_1, \ldots, x_n]$: finite set of variables
- $\mathcal{D}$: a domain for $\mathcal{X}$
- $\mathcal{C}$: finite set of constraints

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- Constraint Satisfaction Problem $(\texttt{CSP})$: deciding whether a constraint network has a solution or not
- CSP is NP-Hard in general

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- Constraint Satisfaction Problem (`CSP`): deciding whether a constraint network has a solution or not
- CSP is NP-Hard in general

- Complete backtracking algorithms

Search & Propagation

Search & Propagation

Search

- Search: decisions to explore the search tree

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- Search in $\mathrm{CP}=$ variable ordering $+$ value ordering

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Search & Propagation

Search

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- Standard or customized

Propagation

- Propagation: inferences based on the current state
- Constraint $\leftrightarrow$ propagator
- The level of pruning $\leftrightarrow$ local consistency

Learning

Learning

$$
\begin{aligned}
&x_1 + x_7 \geq 4 \wedge \\
&x_2 + x_{10} \geq 11 \wedge \\
&x_3 + x_9 = 16 \wedge \\
&x_5 \geq x_8 + x_9 \wedge \\
&b \leftrightarrow (x_9 - x_4 = 14) \wedge \\
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&b \rightarrow (x_6 + x_7 \leq 9) \wedge \\
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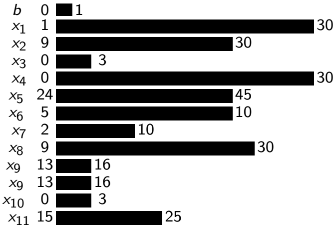

Learning

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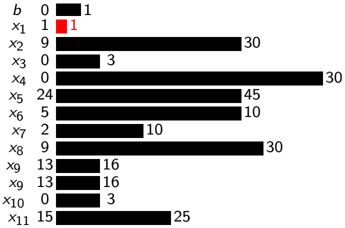
$[\![x_1 = 1]\!]$



Learning

$$[\![x_1 = 1]\!] \longrightarrow [\![x_7 \geq 3]\!]$$

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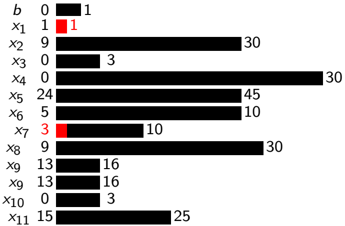

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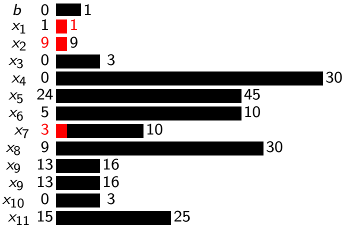

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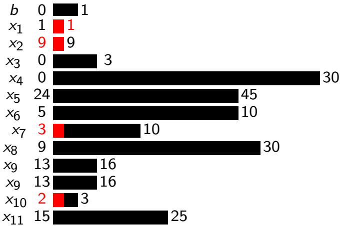

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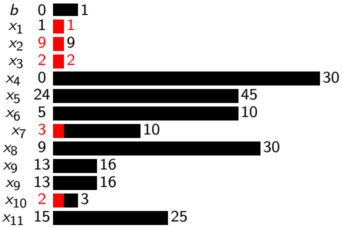

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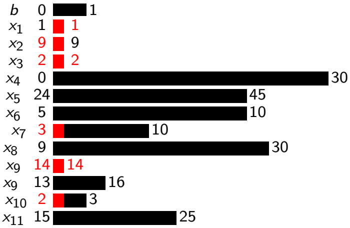

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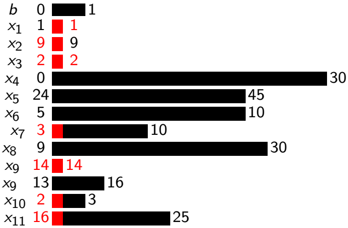

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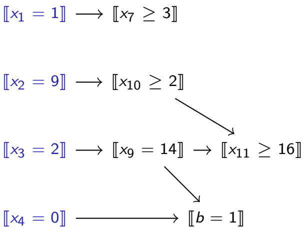

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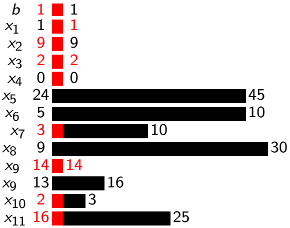

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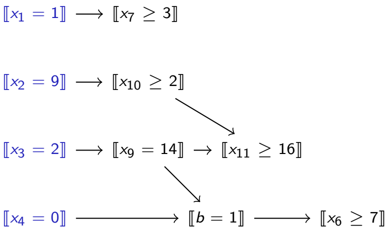

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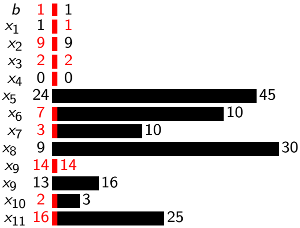

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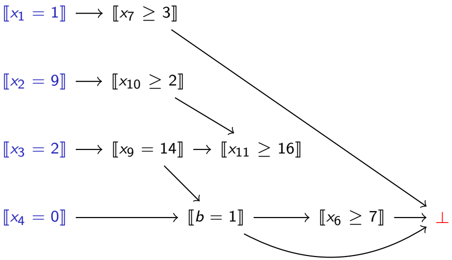

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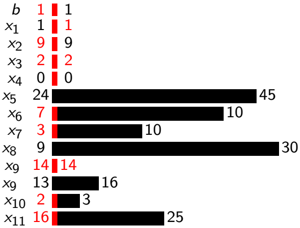

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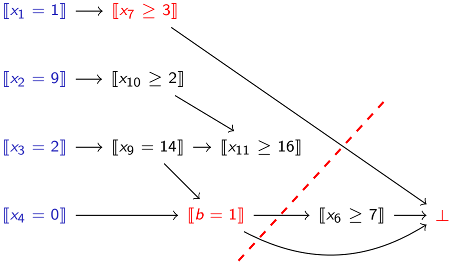

- Conflict analysis: $[\![b = 1]\!] \wedge [\![x_7 \geq 3]\!] \Rightarrow \bot$

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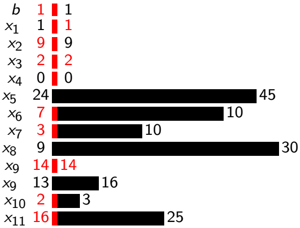

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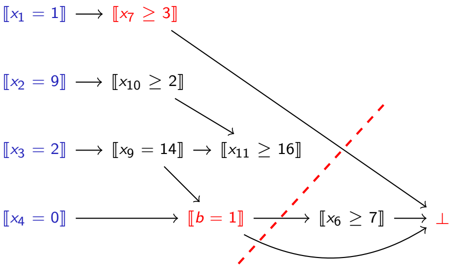

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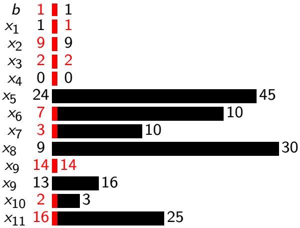

- Conflict analysis: $[\![b = 1]\!] \wedge [\![x_7 \geq 3]\!] \Rightarrow \perp$
- New clause: $[\![b \neq 0]\!] \vee [\![x_7 \leq 2]\!]$

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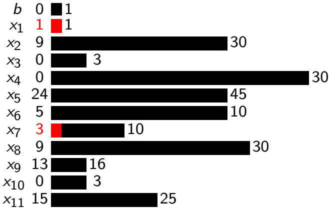

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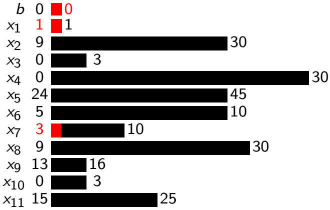
- Propagate the learnt clause



Learning

$[\![x_1 = 1]\!] \longrightarrow [\![x_7 \geq 3]\!] \longrightarrow [\![b = 0]\!]$

- Conflict analysis: $[\![b = 1]\!] \wedge [\![x_7 \geq 3]\!] \Rightarrow \bot$
- New clause: $[\![b \neq 0]\!] \vee [\![x_7 \leq 2]\!]$
- Backtrack to level 1
- Propagate the learnt clause
- Continue exploration

$$
\begin{array}{l}
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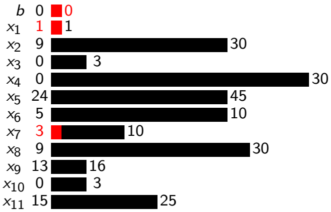
$$



Learning in CP

- Hybrid CP/SAT
- Conflict Driven Clause Learning (CDCL) [Moskewicz et al., 2001]
- Based on the notion of **explanation**

Summary of the thesis

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Modern CP-Solvers may not underestimate any of the three aspects: search, propagation, and learning

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Contributions

- Propagation in a class of sequencing problems

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Contributions

- Propagation in a class of sequencing problems
- Search in car-sequencing

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- Propagation in a class of sequencing problems
- Search in car-sequencing
- Learning in car-sequencing
- Revisiting lazy generation

Summary of the thesis

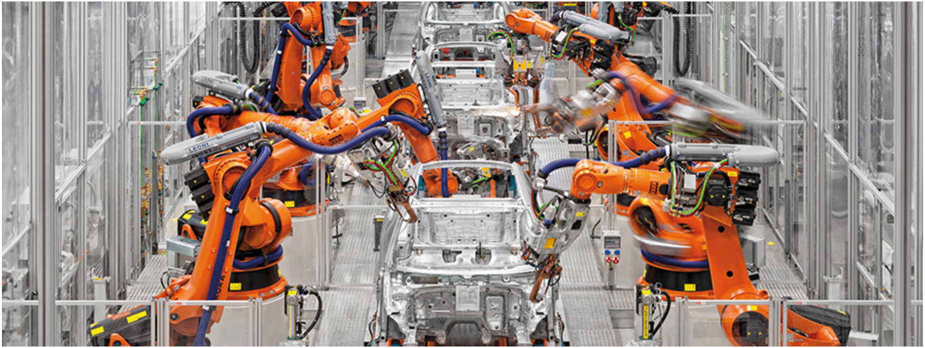
Modern CP-Solvers may not underestimate any of the three aspects: search, propagation, and learning

Contributions

- Propagation in a class of sequencing problems
- Search in car-sequencing
- Learning in car-sequencing
- Revisiting lazy generation
- Learning in disjunctive scheduling

Outline

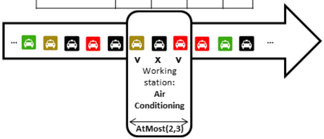
Car-Sequencing



- ROADEF'05 challenge [Solnon et al., 2008]
- RENAULT

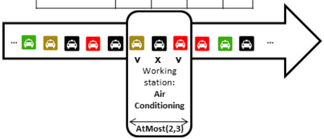
Problem Definition

Air Conditioning	V	X	V	X
Sunroof	X	V	X	V
MP3 Device	V	V	X	X



Problem Definition

Air Conditioning	V	X	V	X
Sunroof	X	V	X	V
MP3 Device	V	V	X	X



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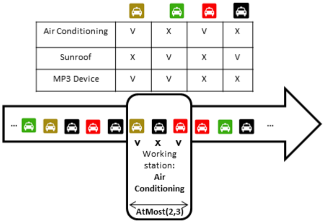

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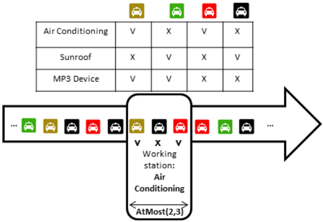

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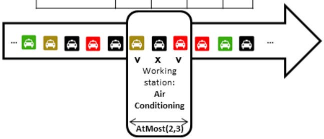

- A class of vehicles is defined by a set of options
- Each class is associated to a demand
- `Capacity` constraints: no subsequence of size q may contain more than p vehicles requiring a given option
- Is there a sequence of cars satisfying both demand and capacity constraints?

Outline

Propagation via AtMostSeqCard

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Definition

AtMostSeqCard$(p, q, d, [x_1, \ldots, x_n]) \Leftrightarrow$

$$\bigwedge_{i=0}^{n-q} \left(\sum_{l=1}^{q} x_{i+l} \leq p\right) \wedge \left(\sum_{i=1}^{n} x_i = d\right)$$

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Example AtMostSeqCard$(2, 5, 4, [x_1, \ldots, x_9])$

x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_9

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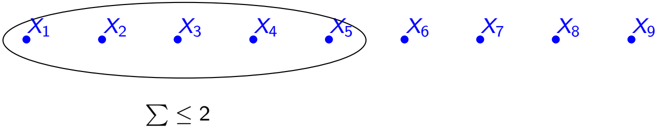

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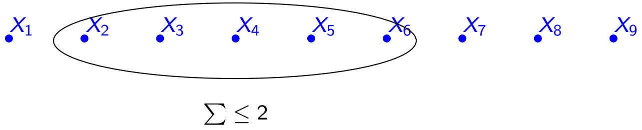

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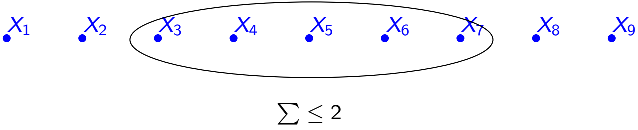

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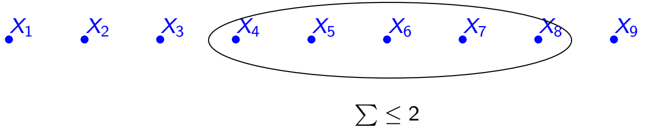

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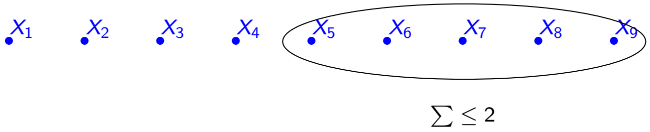

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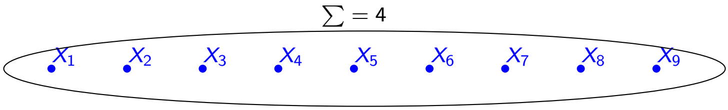

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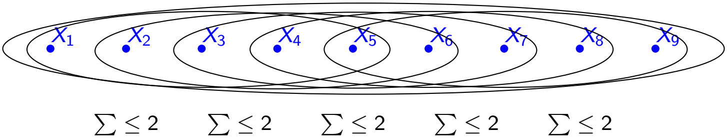
$\sum = 4$



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- Car sequencing
- Crew-Rostering/Timetabling

Arc Consistency on AtMostSeqCard

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A constraint C is Arc Consistent (AC) iff for every x in the scope of C, for every value $v \in \mathcal{D}(x)$ there exists an assignment w in $\mathcal{D}$ satisfying C in which v is assigned to x

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AtMostSeqCard as a particular case?

- cost-Regular: $O(2^q n)$ [van Hoeve et al., 2009]
- Gen-Sequence: $O(n^3)$ [van Hoeve et al., 2009]
- Gen-Sequence: $O(n^2.log(n))$ down a branch $\oplus$ initial compilation of $O(q.n^2)$. [Maher et al., 2008].

Arc Consistency on AtMostSeqCard

An example with $\textsc{AtMostSeqCard}(4, 8, 12, [x_1, \ldots, x_{22}])$

. 0 . . . . . . 0 1 0 1

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- Arc Consistency in $O(n)$ time (optimal)
- Extremely efficient in practice (Car-Sequencing + Crew Rostering)

Outline

Hybrid CP/SAT Models

- Models based on AtMostSeqCard

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- Explain AtMostSeq and Cardinality
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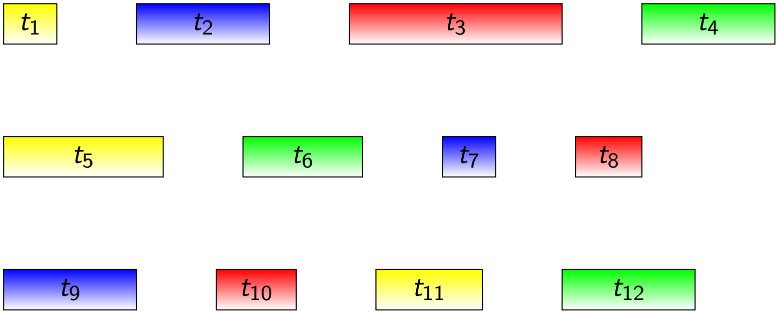
$\widehat{\mathcal{D}}$: 1 1 . . . . . . 1 1 . . . . 0 0 0 0 . 0 0 . . . 1

Experimental Results

- CP, SAT, Hybrid CP/SAT models
- Finding solutions quickly: Propagation is very important to find solutions quickly when they exist.
- For proving unsatisfiability: Clause learning is by far the most critical factor.

Outline

Disjunctive Scheduling



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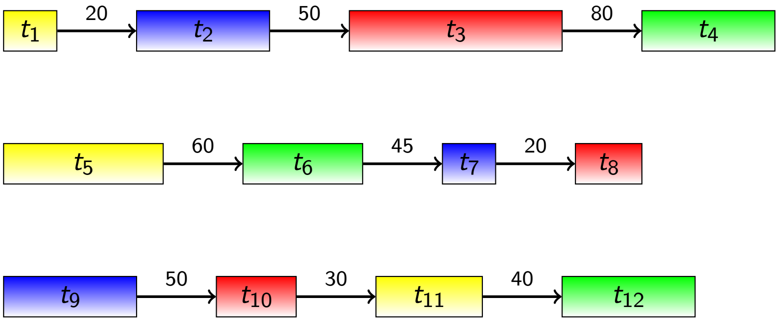

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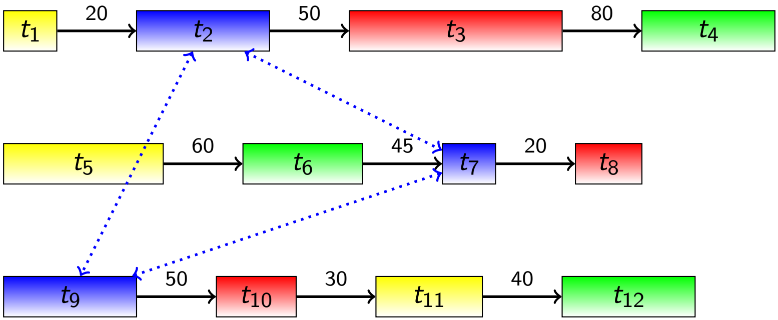

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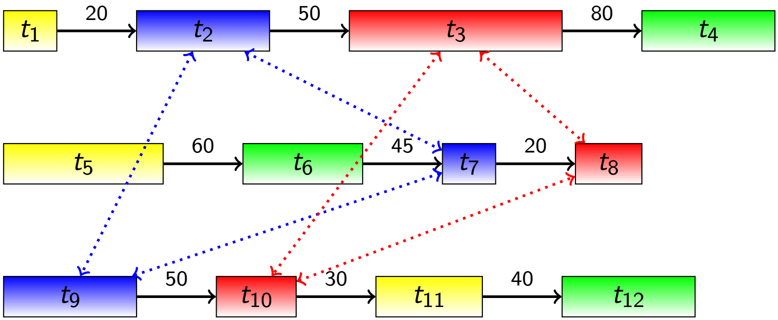

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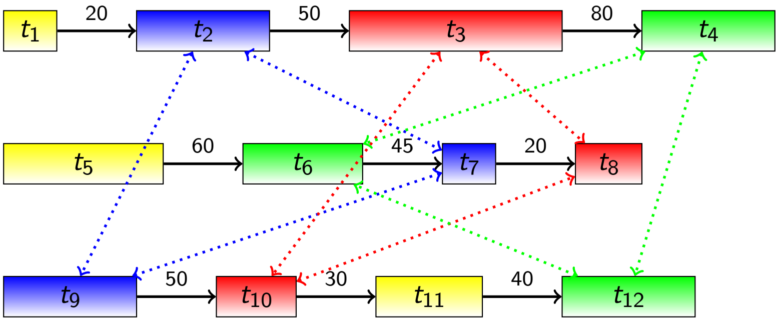

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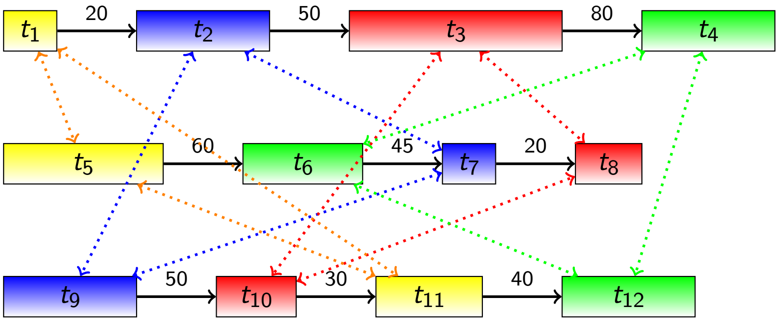

Formulation

Unary Resource Constraint

Formulation

Unary Resource Constraint

- Decomposition using the following DISJUNCTIVE constraints:

$$\delta_{kij} = \begin{cases} 0 & \Leftrightarrow \quad t_{ik} + p_{ik} \leq t_{jk} \\ 1 & \Leftrightarrow \quad t_{jk} + p_{jk} \leq t_{ik} \end{cases} \tag{1}$$

Disjunctive-based Learning

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Conflict analysis in two phases:

1. Standard 1-UIP cut
2. Apply resolution for every bound literal until having a nogood with only reified Boolean variables

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tai13		tai21		tai23		tai25		tai26		tai29		tai30	
new	old	new	old	new	old	new	old	new	old	new	old	new	old
1305	1282	1613	1573	1514	1474	1543	1518	1561	1558	1573	1525	1508	1485

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Summary

- Contributions to each of the three aspects of constraint programming that are ‘search’, ‘propagation’ and ‘learning’ for efficiently solving sequencing and scheduling problems.

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Future Research

- (Car-Sequencing) Application to 'real' industrial situations?
- More extensions for ATMOSTSEQCARD?
- Hand crafted learning?

Thank you for your attention!

Special thanks to my co-authors..

- Christian Artigues
- Emmanuel Hebrard
- Marie-Jose Huguet
- Valentin Mayer-Eichberger
- Nina Narodytska
- Thierry Petit
- Toby Walsh

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